Estimation of the topological state of power system networks via interval analysis

Y. Hassaine, E. Walter, M. Dancre, B. Delourme and P. Panciatici

Abstract—This paper presents a new approach for the estimation of the topological state of power system networks. The quantities to be estimated include Boolean variables indicating the existence or absence of connection as well as real variables. The method is based on interval analysis and the notion of constraint propagation. Discrete variables (status variables) are used to determine a topology. To demonstrate the efficiency of the method, several simulations are conducted on the IEEE 14-bus network.

Index Terms—constraint propagation, hybrid estimation, interval analysis, topology identification.

I. INTRODUCTION

URING the last decade, attention has increasingly been devoted to the problem of topology identification, because topological errors drastically affect the estimates of the real state variables. These topological errors may result from measurement asynchronism or sensor failure. Several methods can be found in the literature for the identification of topology [2], [5] and [6], and particularly noteworthy is the global vision proposed in [7] under the name of generalized state estimation. Most of the methods available attempt to detect and correct topology errors using state estimation residuals. Such an approach fails in the case of interacting data errors. Moreover, local optimization methods are used, and the results obtained are local solutions in the neighborhood of the starting point, without any guarantee of global validity. In this paper, a new approach for topology identification is proposed, which provides guaranteed results in the sense that no solution can be lost. It is based on interval analysis and interval constraint propagation. The paper is organized as follows. Section II illustrates the practical types of problem considered, based on the IEEE

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14-bus network. In Section III, the basic concepts of interval analysis and constraint propagation are introduced and then applied to topology identification. Two generic types of topology identification problems are considered, namely branch and bus topology identification. Finally, Section IV presents the results obtained with this method on the three test cases introduced in Section II.

II. PROBLEM STATEMENT

The three test-cases TC_1 to TC_3 to be considered are all based on the standard IEEE 14-bus network (Fig. 1). More specifically, it is assumed that the part of the network below the transformers on Fig. 1 corresponds to a suspect pocket [7] whose topology is doubtful. The power flows across these transformers are taken as belonging to a 20% interval around the measured value.



Fig. 1. IEEE 14-bus network

In TC_1 the problem is to assess whether there is a branch between buses 2 and 3 assuming that the rest of the

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topology is correctly described. The decision should be based on the measurements of the three active and reactive power flows T_{34} , T_{51} and T_{54} . In TC_2 it is feared that the same branch between buses 2 and 3 as in TC_1 may actually be connecting buses 3 and 4, in parallel with an already existing branch. Decision on the topology should be based on voltage measured at buses 2 and 3 and on active and reactive power flows T_{34} and T_{12} . In TC_3 , bus 2 may either actually correspond to a single bus or be split into buses 2_1 and 2_2 . In the latter case, bus 2_1 is connected to buses 1 and 4, and bus 2_2 to buses 3 and 5. Decision should be based on active and reactive power flows T_{23} and T_{25} . Moreover, the branch l_3 originating from bus 2 (or 2_1) may either be connected to bus 4 or disconnected. For each of these test-cases, the relative precision of voltage measurements is 5% and that for active and reactive power flows measurements is 20%. The prior domains for all non-measured variables are computed as follows. The power flow T_{ij} from bus *i* to bus *j* is assumed to belong to the interval $\left[-2y_{ij}V_n^2, 2y_{ij}V_n^2\right]$, where V_n is the nominal voltage magnitude, and y_{ij} is the modulus of the admittance of the branch connecting buses i and j, both given. Active injections are assumed to be within 20% of forecasted values. No information is available about non-measured reactive injections, which implies that the prior domains for these variables are taken equal to $[-\infty,\infty]$. It is also assumed that the difference of the phases between the two extremities of any branch is smaller than 12° . Finally, voltages are assumed to be within 15% of their nominal value.

 TC_1 and TC_2 are representative of branch topology identification, to be considered in more detail in Section III.C.2, whereas TC_3 mixes bus and branch topology identification. Bus topology problems will be treated in Section III.C.3. Each of these test-cases will be solved using a new approach based on interval analysis, which will now be presented before returning to them.

III. INTERVAL FORMULATION

A. Basic concepts

Interval analysis (sec, e.g., [4] and [8]), makes it possible to derive *guaranteed* numerical algorithms for the solution of mathematical problems. Guaranteed means here that no solution can be lost. Outer (and sometimes inner) approximations of the solution set can be obtained. This solution set is not required to be convex or even connected for the methodology to apply. All quantities are assumed to belong to intervals, which describe their degree of uncertainty. Deterministic quantities will be represented by degenerate intervals with zero width. Completely unknown quantities will be assumed to belong to the interval $[-\infty, \infty]$.

It is a simple matter to extend all classical operations on real numbers to intervals. Let \underline{x} and \overline{x} be the lower and upper bounds of the interval [x]. The addition of two intervals, for instance, is defined as follows

$$[x] + [y] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]. \tag{1}$$

Multiplication, division, union and intersection of two intervals can similarly be defined. All elementary functions operating on real numbers can also be extended to intervals. For instance $\exp([x]) = [\exp(\underline{x}), \exp(\overline{x})]$.

This paper will show how interval analysis can be used to compute intervals guaranteed to contain the possible values of all uncertain variables. Computation will take advantage of (i) prior information, (ii) the experimental data and (iii) the structure of the mathematical model under consideration.

Consider a vector **x** comprising *n* variables $x_i \in \mathbb{R}$, i = 1, ..., n. Assume that these variables are linked by *m* equality constraints

$$f_i(x_1, ..., x_n) = 0, \ j = 1, ..., m.$$
 (2)

These constraints can be expressed in vector form as

$$\mathbf{f}(\mathbf{x}) = \mathbf{0}.\tag{3}$$

Each variable x_i belongs to some prior domain, which will here be assumed to be an interval for simplicity. Thus, the prior domain for \mathbf{x} is a vector interval (or *box*), denoted by $[\mathbf{x}]$. Looking for the set S of all \mathbf{x} in $[\mathbf{x}]$ such that (3) is satisfied is a *constraint satisfaction problem* (CSP), concisely denoted by

$$H: (\mathbf{f}(\mathbf{x}) = \mathbf{0}, \mathbf{x} \in [\mathbf{x}]). \tag{4}$$

Interval analysis uses a contractor to replace the prior box $[\mathbf{x}]$ by a smaller box $[\mathbf{x}']$ such that the solution set remains unchanged $\mathcal{S} \subset [\mathbf{x}'] \subset [\mathbf{x}]$.

A possible method to build contractors is via interval constraint propagation.

B. Interval constraint propagation

To understand the basic idea of interval constraint propagation, consider the box $[\mathbf{x}] = [2,4] \times [2,4] \times [12,24]$,



Fig. 2. Action of a contractor

and assume that its components are linked by the constraint $x_3 = x_2 x_1$. This constraint can also be written as $x_1 = x_3/x_2$ or $x_2 = x_3/x_1$. Each of these relations has been obtained by isolating one of the variables in the initial expression.

Since $x_1, x_2 \in [2,4]$ and $x_3 \in [12,24]$ then $x_3/x_2 \in [12,24]/[2,4] = [3,12]$. As $x_1 = x_3/x_2$ and $x_1 \in [2,4]$, then $x_1 \in [3,12] \cap [2,4] = [3,4]$. The same operations are performed on the other variables x_2 and x_3 , and the resulting contracted domains for x_1, x_2 and x_3 are [3,4], [3,4] and [12,16].

The contraction above can be described by the following equations

$$\begin{cases} [x_1] = [x_1] \cap ([x_3]/[x_2]), \\ [x_2] = [x_2] \cap ([x_3]/[x_1]), \\ [x_3] = [x_3] \cap ([x_1][x_2]), \end{cases}$$
(5)

which may be applied as long as contraction is taking place.

C. CSP for topology identification

1) Network equations : Interval analysis and the notion of CSP are now applied to topology identification as part of the generalized state estimation problem. It is assumed that a suspect pocket has been isolated and that the suspect branches and buses have been localized inside this suspect pocket, using for example the method suggested in [2], [5] and [6].

Two types of problems will be considered, namely the identification of branch topology and of bus topology. Let T_{ii}^{a} and T_{ii}^{r} be the active and reactive power flows from

bus i to bus j. They satisfy

$$\begin{cases} T_{ij}^{a} = V_{i}V_{j}(S_{ij}G_{ij}^{c} - G_{ij}^{s}C_{ij}) + V_{i}^{2}G_{ij}^{s}, \\ T_{ij}^{r} = V_{i}V_{j}(S_{ij}G_{ij}^{s} - G_{ij}^{c}C_{ij}) + V_{i}^{2}(G_{ij}^{c} - H_{ij}), \end{cases}$$
(6)

with

$$\begin{cases}
C_{ij} = \cos(\theta_i - \theta_j), \\
S_{ij} = \sin(\theta_i - \theta_j), \\
G_{ij}^s = y_{ij}\sin(\xi_{ij}), \\
G_{ij}^c = y_{ij}\cos(\xi_{ij}),
\end{cases}$$
(7)

where V_i is the voltage magnitude at bus i, θ_i the voltage phase angle at bus i, $y_{ij} \exp^{-j\xi_{ij}}$ the inductive admittance of the branch connecting buses i and j, and H_{ij} the shunt susceptance of this branch. Using C_{ij} and S_{ij} as uncertain variables instead of θ_i and θ_j as indicated in (7) drastically simplifies interval computations, as the evaluation of trigonometric functions is thus avoided.

To take into account the properties of the sine and cosine functions, one must add the following constraints in the CSP

$$\begin{cases} S_{ij}^{2} + C_{ij}^{2} = 1, \\ C_{ij} = C_{ji}, \\ S_{ij} = -S_{ji}. \end{cases}$$
(8)

2) Identification of branch topology : The most basic topology problem faced by the operators is the identification of branch topology. In such a problem, one of the branches originating at bus n ends at some unknown location to be identified from a set of possible buses. Each of the possible terminal buses n_i of a suspect branch l is associated with a binary status coefficient α_{li} . This coefficient is equal to zero if the bus n_i is not connected to the branch l, and to one if it is connected. Since the branch l is only connected to one terminal bus, only one of the α_{li} differs from zero. Let \mathcal{I}_l be the set of the indices of all buses that may be connected with the bus n via the suspect branch l. A disconnected branch can be modeled by introducing a fictitious bus n_0 characterized by zero active injection, $T_{n_0}^a = 0$, with $\alpha_{ln_0} = 1$. In this case, the index 0 will be introduced in \mathcal{I}_l . To guarantee that a unique α_{li} is equal to one, all the others being zero, the following constraints are included in the CSP

$$\sum_{i \in \mathcal{I}_l} \alpha_{li} = 1, \tag{9}$$

$$\forall (i,j) \in \mathcal{I}_l^2, \ i \neq j: \ \alpha_{li}\alpha_{lj} = 0.$$
 (10)

With this notation, the power injection at bus n is

$$I_n = \sum_{j \in \mathcal{S}_n} T_{nj} + \sum_{l \in \mathcal{L}_n} \sum_{i \in \mathcal{I}_l} \alpha_{li} T_{ni}, \qquad (11)$$

where \mathcal{L}_n is the set of the indices of the suspect branches originating at bus n, and \mathcal{S}_n is the set of the indices of

the terminal buscs of the reliable branches originating at bus *n*. Fig. 3 describes the situation in the simple case $\mathcal{L}_n = \{1\}, \mathcal{I}_l = \{0, 1, 2, 3, 4\}$ and $\mathcal{S}_n = \{m\}$.



Fig. 3. Problem of branch topology; only one of the α_{Ii} is equal to one, all the others are zero

3) Identification of bus topology : A more complex topology problem is to allocate buses correctly inside a substation. Let \mathcal{K} be the set of the indices of the suspect substations. In the suspect substation indexed by $k \in \mathcal{K}$, the maximum number of buses is N_k . For the sake of simplicity, it will be assumed that only one branch topology is associated with the n-th possible bus topology, where $n = 1, ..., N_k$. If there are more than one possible branch topology in the n-bus case, then the problem to be considered becomes a combination of branch and bus topology problems (see TC_3). For each bus $i \ (i < n)$ of an *n*-bus substation, the corresponding injection is denoted by I_{ni}^k , and the set of the coefficients of the connected branches is denoted by \mathcal{M}_{ni}^k . Again, the binary status coefficient α_{kn} takes the value one if the substation indexed by k involves n buses and zero otherwise. With this notation, the constraints can be written as

$$\alpha_{k1}(I_{11}^{k} - \sum_{m \in \mathcal{M}_{11}^{k}} T_{m}) = 0,$$

$$\begin{cases} \alpha_{k2}(I_{21}^{k} - \sum_{m \in \mathcal{M}_{21}^{k}} T_{m}) = 0, \\ \alpha_{k2}(I_{22}^{k} - \sum_{m \in \mathcal{M}_{22}^{k}} T_{m}) = 0, \\ \vdots \\ m \in \mathcal{M}_{n1}^{k} - \sum_{m \in \mathcal{M}_{n1}^{k}} T_{m}) = 0, \\ \vdots \\ \alpha_{kn}(I_{n1}^{k} - \sum_{m \in \mathcal{M}_{n1}^{k}} T_{m}) = 0, \\ \vdots \\ \alpha_{kn}(I_{n1}^{k} - \sum_{m \in \mathcal{M}_{n1}^{k}} T_{m}) = 0, \\ \vdots \\ \alpha_{kn}(I_{n1}^{k} - \sum_{m \in \mathcal{M}_{n1}^{k}} T_{m}) = 0, \end{cases}$$
(12)



Fig. 4. topology bus problem; the single bus n of the upper diagram is split into the buses n_1 and n_2 of the lower diagram

 $k \in \mathcal{K}, i \leq n \in \{1, ..., N_k\}$. For any $n \in \{1, ..., N_k\}$, the goal is to find the single α_{kn} that differs from zero.

The next two constraints are added to guarantee that the solution for the status coefficients is unique

$$\sum_{n=0}^{N_k} \alpha_{kn} = 1, \tag{13}$$

$$\forall (n,m) \in \{1,...,N_k\}^2, \ n \neq m : \ \alpha_{kn} \alpha_{km} = 0.$$
 (14)

Fig. 4 illustrates a simple case where $\mathcal{K} = \{1\}, N_1 = 2$ and

$$\begin{cases} \mathcal{M}_{11}^{1} = \{1, 2, 3, 4\} \\ \mathcal{M}_{21}^{1} = \{1, 2\} \\ \mathcal{M}_{22}^{1} = \{3, 4\} \end{cases}$$
(15)

Equation (12) then becomes

$$\begin{cases} \alpha_{11}(I_{11} - (T_1 + T_2 + T_3 + T_4)) = 0, \\ \alpha_{12}(I_{21} - (T_1 + T_2)) = 0, \\ \alpha_{12}(I_{22} - (T_3 + T_4)) = 0. \end{cases}$$
(16)

In order to solve (16) using constraint propagation, and to avoid numerical instabilities, additional information regarding the injections and available in the data base must be taken into account. For this example, $I_{11} = I_{21} = I$ and $I_{22} = 0$, and (16) becomes

$$\begin{cases} I - (T_1 + T_2) - \alpha_{11}(T_3 + T_4) &= 0, \\ \alpha_{12}(T_3 + T_4) &= 0. \end{cases}$$
(17)

4) Guaranteed generalized state estimation: The model equations can be put in vector form as

$$[\mathbf{z}] = [\mathbf{h}]([\mathbf{v}], [\boldsymbol{\alpha}], [\mathbf{C}], [\mathbf{S}]), \tag{18}$$

where $[\mathbf{z}]$ is the real interval vector of the power flows and injections, $[\mathbf{v}]$ is the real interval vector of the voltage magnitude moduli, $[\mathbf{C}]$ and $[\mathbf{S}]$ are the real interval matrices of the sines and cosines of the phase shifts and $[\boldsymbol{\alpha}]$ is the interval matrix of status coefficients. The prior intervals for each status coefficient is [0, 1], and it is expected to be contracted into the degenerate interval [0, 0]or [1, 1] when solving the associated CSP.

The description of the three test cases in Section II indicates how the other prior intervals are computed. The approach advocated in this paper is to contract the prior domains for all variables until a decision is reached regarding the value of the Boolean status coefficients or no further contraction can be obtained.

IV. RESULTS

This approach has been implemented using the interval solver *RealPaver* [3]. RealPaver contracts the prior domains for [z], [v], [α], [C] and [S], and generates a sequence of boxes guaranteed to contain the set S of all solutions to (18). For TC_1 and TC_2 there is only one suspected topological problem, and a single set of α variables can be used, whereas TC_3 , which combines two types of topological problems, will require two sets of variables α and β .

For TC_1 , the problem is to determine if buses 2 and 3 are connected as indicated on Fig. 1 ($\alpha_1 = 1$) or disconnected ($\alpha_0 = 1$). The prior domain for each of the status variables α_i is taken as [0, 1]. The contracted domains for these status variables are as indicated in Table I, and the estimator thus concludes that buses 2 and 3 are indeed connected.

For TC_2 , the branch indicated on Fig. 1 as connecting buses 2 and 3 may indeed connect these buses $(\alpha_1 = 1)$ or actually connect buses 3 and 4 in parallel with the existing branch structure of $(\alpha_2 = 1)$. Again, the prior domain for each of the status variables α_i is taken as [0,1]and the contracted domains for these status variables are as indicated in Table I. The estimator concludes that the structure of Fig. 1 is correct, with a branch connecting buses 2 and 3.

Finally, for TC_3 , two problems are addressed simultaneously. The first one is deciding whether bus 2 is indeed

a single bus $(\alpha_1 = 1)$ or should be split into buscs 2_1 and 2_2 as explained in Section II $(\alpha_2 = 1)$. The second one is to decide whether the same suspect branch as in TC_1 originating at bus 2 (or 2_1) is actually connected at bus 3 $(\beta_1 = 1)$ or disconnected $(\beta_0 = 1)$. As usual, the prior domain for each of the status variables α and β is [0, 1]. The contracted domains for these status variables are as indicated in Table I, and the estimator thus concludes that bus 2 should not be split and that buses 2 and 3 are connected.

TABLE I STATUS COEFFICIENTS FOR TC_1 , TC_2 and TC_3

	Prior intervals	Results
TC_1	$lpha_0=[0,1]$	$\alpha_0 = 0$
	$lpha_1=[0,1]$	$\alpha_1 = 1$
TC_2	$lpha_1=[0,1]$	$\alpha_1 = 1$
	$\alpha_2 = [0, 1]$	$\alpha_2 = 0$
TC_3	$lpha_1=[0,1]$	$\alpha_1 = 1$
	$lpha_2=[0,1]$	$\alpha_2 = 0$
	$eta_0=[0,1]$	$\beta_0 = 0$
	$\beta_1 = [0, 1]$	$\beta_1 = 1$

A few observations are in order. First, the intervals [C] and [S] relative to the phases are more contracted than the voltage intervals $[\mathbf{v}]$. This is not surprising given the greater number of constraints (8) on [C] and [S]than on [v]. RealPaver takes advantage of the reduction of $[\mathbf{C}]$ and $[\mathbf{S}]$ to contract the interval status coefficients $[\alpha]$ and $[\beta]$. As soon as the prior interval domain for a status coefficient is contracted, this coefficient becomes certain, because of the Boolean nature of the associated variable. Secondly, the physics of the network lead to an active-reactive decoupling [1] and a dependance between voltage phase shift and active measurements. Therefore, the topology identification is more sensitive to active measurements than to reactive ones. This qualitative analysis is corroborated by the identification of the correct topology in TC_3 using the active measurements only. To reach an unambiguous conclusion when attempting to identify the topology of a network, the localization and accuracy of the measurements are more important than their number. This is connected to the notion of criticity, which could be studied via a specific type of redundancy analysis that remains to be developped.

Finally, to accelerate contraction and increase the accuracy of the results, it is interesting to introduce as many pertinent constraints as possible. For instance, the following constraint can be added

$$T^a_{ij}T^a_{ji} \le 0, \tag{19}$$

to guarantee that the active power flows at the endpoints of a branch are of opposite signs. A more accurate constraint could be obtained by taking into account the maximum loss in each branch.

V. CONCLUSIONS

A new method to identify the topology of a suspect pocket in a power system network has been proposed. Interval analysis and constraint propagation are used to implement generalized state estimation in a guaranteed way. As the results from the standard IEEE 14-bus network tend to show, the method is promising and makes it possible to identify the bus and branch topologies in suspect pockets. Other applications to the identification of parameters in power system networks should be investigated in the near future, such as the identification of transformer tap positions.

VI. References

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VII. BIOGRAPHIES

Yacine Hassaine was born in Algiers, Algeria, on December 1977. He graduated from École National Polytechnique d'Alger and he is preparing a Ph. D in state estimation and topology identification with Réseau de Transport d'Électricité (RTE, the French transmission system operator) and the Laboratoire des signaux et systèmes, CNRS-Supélec-Université de Paris Sud.

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Mathieu Dancre was born in Neuilly sur Marne on January 1974. He graduated from the École Nationale Supérieure de Physique de Marseille (1996) and received a Ph. D (1999) in computer science from the University of Marseille (France). Since 1999, he has been working with EDF R&D on EMS functions of power systems, focusing research mostly on state estimation.

Benoit Delourme was born in Rennes, France, on December 1977. He graduated from École Supérieure d'électricité Supélec in 2000. He was with EDF and Hydro-Quebec in 2000-2002 and worked on transient analysis of power systems. He joined RTE (French transmission system operator) in 2002. His research now is devoted to state estimation and optimal power flow.

Patrick Panciatici was born in March 1960, graduated from École Supérieure d'électricité Supélec in 1984. He joined EDF R&D in September 1985. Since 1998, he is the head of a team which develops Security Analysis tools for real time (EMS) and operational planning now at RTE (French transmission system operator). During the last 17 years, he has contributed in different projects at EDF R&D : Coordinated Secondary Voltage Control (CSVC), Power System Simulation (EUROSTAG) and Short Term Load Forecast.