# TOLERANCE DESIGN OF DC-DC SWITCHING REGULATORS

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Abstract. *Optimized tolerance design of dc-dc switching regulators is discussed in this paper. The tolerance design of analog control feedback compensators for dc-dc regulators is considered. Given the desired tolerances on dynamic performances of the closed loop regulator, in terms of crossover frequency and phase margin, a Genetic Algorithm (GA) seeks the set of values of parameters with maximum tolerance, falling within a Region Of Acceptability (ROA) determined by means of interval analysis (IA). The joined GA-IA approach is applied to the design of a voltage-mode controlled regulator.*

## I. Introduction

Tolerance design is one of the most attractive challenges in modern circuit design and especially in power electronics. Tolerance design, usually following the nominal design, is mandatory to ensure the robustness of the design with respect to parameters' drifts due to thermal effects, aging and so on. The effects of these variations are rapidly becoming as much critical as the physical dimensions reduce and customer' specifications are more stringent. Moreover, the two-steps nominal and tolerance design not always leads to the optimal circuit design. It should be desirable, instead, to perform these two actions jointly, thus placing the nominal design where the minimum sensitivity with respect to parameters is ensured. Performing the nominal design of the network and, at the same time, determining the maximum deviation of the parameters from their central values that guarantee performance specifications, allows to maximize the yield. In addition, per-unit circuit cost is reduced and the robustness of the design with respect to a further parameter variation is detected. These analyses are often performed accounting for the components' tolerances only, thus considering a normal distribution of parameters' values within their respective intervals. Nevertheless, whenever high reliability is required in circuit design, it must be assumed that the variations of the parameters between their extreme values are uniformly distributed and independent one from the others. This leads to the True-Worst-Case (TWC) Tolerance Design (TD), which requires additional costs in term of circuit simulations. A further element that increases the difficulty of the design is the need of a 100% yield, required either by the customer or by the application. This is given whenever the circuit cost is a heavy influential factor, as in custom, military and space applications. These elements concur to complicate the designer's work in searching for the optimal design of the circuit. The design specifications play a fundamental role in the feasibility of the final design and in the effort required to achieve it. Specifications can be translated in the N-dimensional space of parameters as surfaces: the resulting N-dimensional region is called Region of Acceptability (ROA) that may be non-convex and not simply connected. The unpredictable characteristics of the ROA make difficult the application of some techniques for tolerance design [1] based on a Monte Carlo (MC) sounding of the TR. Such method ensures the dimensional independence [1], but requires a high computational cost and involves an unacceptable uncertainty on the resulting value of the yield. In this paper the tolerance design of a control feedback compensation network for a dc-dc voltage regulated switching converter is considered. This has been taken as a testing bench of the TWC tolerance design and yield calculation techniques presented in [3]. In this paper nominal and tolerance designs are performed jointly by means of a Genetic Algorithm (GA). Using the vertex analysis [1], the nominal values and tolerances of the circuit parameters are tuned to get a 100% yield by sounding the space of parameters, discretized according to commercial sizes. Since the tolerance region is checked in its vertex only, this approach guarantees an optimal design, but it might not ensure the 100% yield if the ROA is non-convex and/or not simply connected. To validate the result and to evaluate the actual yield, the Interval Analysis (IA) based technique presented in [2] is applied. The method is robust and gives the yield value with the required accuracy regardless of the geometrical characteristics of the ROA. The design process of the compensation network is performed accordingly to the following steps:

- the optimum yield design is carried out by means of the GA-based algorithm, which maximizes the tolerances of the designed circuit parameters;
- the project is corrected using the commercial values of parameters closer to the optimum designed ones and is tested by means of the IA-based algorithm illustrated in [2], which calculates the true yield of the commercial project.

## II. The EA-Based Approach

GA's are based on the principles of biological evolution [3] and are often used as optimizers, as they ensure a better balance between the need of an exhaustive visit of the space of the solutions and the will of tracking "good" solutions toward the search of the best one. The crossover and mutation operators exert an evolutionary pressure on a population of individuals, each one representing a possible solution of the given problem. The constrained optimization problem in subject has been formulated as an unconstrained one thanks to the use of a non-uniform penalty function. The penalty weight multiplied by the objective score of unfeasible individuals gives their fitness score. It is very lenient at the beginning and increases its harshness as the algorithm runs. This approach drives the algorithm towards clusters of feasible solutions, while preserving in the mating pool some unfeasible individuals that can be closer to the optimum than many feasible ones. The opportunity this approach offers to navigate through unfeasible regions of the search space allows giving better final individuals than the approaches limiting search paths to feasible regions only.

*The chromosomes' structure.*

A real-coded representation of the genes has been adopted [3], with each individual identified by as many couples of central values and tolerances as the number of the parameters to be designed. To improve the search capabilities of the algorithm

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without impoverishing the results of the design process, the range of central values allowed for each parameter, as much as the tolerances, have been restricted to a reasonable interval of the N-dimensional real numbers space. Then, upper and lower bounds for both central values and tolerances are fixed at the beginning of the GA. This allows to perform a true search of the optimal set of parameters' central values/tolerances, in contrast with the usual tolerance assignment problem starting from a pre-determined "nominal" design point in the space of parameters. Moreover, to give a practical meaning to the central values/tolerances determined by the optimization process, the search space has been discretized with a resolution depending on the less significant digit required. Each set of parameters' central value/tolerance $[p_1^0, p_2^0, \ldots, p_{N-1}^0, p_N^0, t_1, t_2, \ldots, t_{N-1}, t_N]$, corresponding to an hyper-rectangle with $2^N$ vertices in the space of parameters, represents a real-valued chromosome of the GA.

*The chromosome fitness.*

The objective function, whose value gives the goodness of each individual, can be chosen between the desired circuit and/or components cost functions. Unfeasible individuals are penalized and are included in the mating pool for the next generation only in case there were less feasible individuals than the total number of individuals of the population. The fitness score of the k-th individual is $\varphi(k)=f(k)\cdot e^{p\alpha\tau}$, where f(k) is the objective score (fitness) and the parameters of the exponential-shaped penalty function, or attenuation factor are user-defined: $\alpha$ is a penalty constant, positive for minimization problems and negative for maximization ones, $\tau \in [0,T]$ is the current generation, T is the total number of generations of the run, $p \in [0,2^N]$ is an integer number that gives a measure of the unfeasibility of the individual, its value being 0 if the genome has all the vertex lying in the ROA and $2^N$ if all the vertex are out of the ROA. The exponential penalty term reduces the probability of reproduction of an individual as much as higher the number of its unfeasible vertices is. The integer variable $\tau$ ensures an increasing penalization of unfeasible individuals as the evolution goes on. The unfeasible individuals survive for as few generations as higher $\alpha$ is; therefore $\alpha$ is used to settle the incidence of the penalizing exponential term during the evolution. A high value of $\alpha$ can be useful when the GA is able to pick some feasible individuals in few generations; otherwise, this choice can greatly compromise the convergence of the algorithm. On the contrary, a low value of $\alpha$ can reduce too much the evolutionary pressure towards the optimal individual. For the problem under study, the GA results show low sensitivity in a wide range of $\alpha$.

## III. AN EXAMPLE

Fig.1 shows a compensation network that can be used for analog feedback control of dc-dc switching converters [4]. It is one among several possible design solutions, as illustrated in the paper [5]. The GA-based algorithm for tolerance design has been used for the TWC TD of the compensation network of fig.1 under the two different design constraints expressed by the ranges reported in Tab.I. The two cases express different levels of constraining, the first one more stringent than the second one. GA's settings are shown in Tab.II. The left part of Tab.III shows the bounds and the level of discretization of the subset of the space of parameters wherein the optimal design solution has been searched. The parameters $R_5$ and $R_6$ are fixed at the nominal value: $R_5$=20kΩ and $R_6$=10kΩ [5]. The product of the parameters' percent tolerances has been considered as the objective function to be maximized.

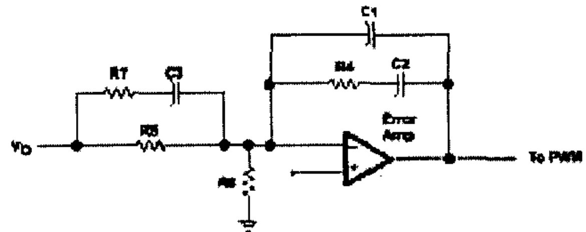


Fig.1. Compensation network

Table I: constraints imposed to the magnitude and phase of the frequency response of the network of fig.1.

| | Crossover Frequency | Phase Margin |
|---|---|---|
| Case 1 | [9,11] kHz | [45°,60°] |
| Case 2 | [6.7,20] kHz | [45°,90°] |

Table II: GA settings.

| | |
|---|---|
| Number of individuals | 60 |
| Number of generations | 200 |
| Rate of population replacement | 90% |
| Probability of crossover | 60% |
| Probability of mutation | 10% |
| Penalty function weight ($\alpha$) | 0.05 |

Figs.2-4 show the evolution of parameters and related tolerances in case 1 of Tab.I vs the number of generations. The plots highlight the visit of the space of solutions. The final (presumed) 100% yield results reported in Tab.3 show that the circuit has an intrinsic lower sensitivity with respect to capacitances C1 and C2, as they saturate in both cases the maximum allowed tolerance without leading to unfeasible solutions. The other parameters, instead, are constrained to smaller tolerances. In case 1 design of Tab.I, the ideal couples of nominal value and tolerance for the five circuit parameters given by the GA-TD are:

| $C_1$[pF] | $C_2$[nF] | $C_3$[nF] | $R_4$[kΩ] | $R_7$[Ω] |
|---|---|---|---|---|
| 6.4±10% | 75.2±10% | 54.4±7.5% | 20±5% | 420±7.5% |

The IA-based algorithm [2] run on the ideal set of parameters' values confirmed the 100% yield assumed by the EA-based algorithm. The ratio between the volume of the smallest subset the procedure must check and the volume of the whole tolerance region to be inspected has been used as accuracy probe for the IA-based algorithm for yield calculation. To enlighten the computation time of the IA based procedure, an MC analysis has been performed over the undetermined subsets after the maximum fixed level of interval partitioning has been overcame. With an accuracy of $10^{-2}$, the 100% yield verification required $10^5$ evaluations of the set of constraints, 80% by IA and 20% by MC computations. This gives also an indication about the position of the tolerance region given by the GA-based tolerance design algorithm within the ROA. The 100% feasibility indicates that the optimal tolerance region determined is fully included within a simply connected and convex ROA.

The second step of the design procedure has been started using the results of GA-TD. The ideal-values of the control feedback compensation network determined by means of the GA-TD have been converted in commercial-values. The resulting commercial-values-tolerances-based tolerance region is then validated by means of the IA based technique presented in [2]. Such analysis allows determining the feasibility and the yield of the compensation network realized by commercially available components with selected couples of nominal value and tolerance. Among the possible sets of commercial values closest to those ones calculated by the GA, the following one has been considered:

| $C_1$[pF] | $C_2$[nF] | $C_3$[nF] | $R_4$[kΩ] | $R_7$[Ω] |
|---|---|---|---|---|
| 6.0±5% | 68.0±10% | 47.0±10% | 20±5% | 430±5% |

Running the IA program gives an 89.3% yield. As the yield is less than 100%, the commercial tolerance region crosses the ROA boundary somewhere. As a consequence, this required the double of the evaluations of the parameters' set with respect to the ideal solution. This result can be read out under different points of view. During the analysis driven by the IA-based method [2], the unfeasible subsets of the tolerance region under test can be saved and used to optimally trimming some commercial values. This operation, which can be automated at a low computational cost, leads to the exclusion of such subsets from the tolerance region, and then to a higher yield. The second step of the design procedure described in this paper may thus become an integral part of the design procedure, not only a tool to confirm and validate the results given by the GA-based algorithm. The second way of using the IA-based verification relies on its robustness in guaranteeing the reliability of results at reasonable computational cost. It allows to quickly test the limited number of possible commercial sets of parameters closest to the optimal GA-determined set, thus enabling the designer to choose the best design set in terms, for example, of cost and/or robustness. To highlight a further feature offered by the IA-based yield calculation algorithm, only two parameters of the five included in the designed set have been left spanning a wide interval centered in their nominal value, while the other three parameters have been fixed at their nominal values. This is a possible way to make a sliced mapping of the ROA in planes of interest to the designer. Some examples of results obtained with this analysis are reported in fig.4. Using such graphs, the designer may optimally move the tolerance region of the designed parameters to improve the yield. This operation can be performed in the parameters' space without going through the performances' space back and forth. Fig.4 also puts in evidence the refinement the IA-based algorithm makes across the borders of the ROA, which are included in the tolerance region in the examples of fig.4. Quickly conquering regions fully in or fully out of the ROA greatly helps the designer in placing the design tolerance region. Such a kind of intelligence, characterizing the method proposed in this paper, can be used efficiently in combination with basic MC analysis.

## IV. CONCLUSIONS

In this paper the tolerance design of a feedback compensation circuit used for control of dc-dc switching converter has been performed by means of two step approach. A genetic algorithm is used to optimally place the tolerance region within the region of acceptability by means of vertex analysis. Such TWC design is oriented to a (presumed) 100% yield and to maximizing the components' tolerances. The second step of the design procedure validates the set of commercial values of parameters closer to the ones provided by the ideal optimum design. This analysis is performed by means of an IA-based algorithm that allows determining, with the desired accuracy, the yield of the commercial design set. It also gives useful information on how to improve the quality of the project in terms of circuit's cost and/or robustness with respect to parameters drifts.


## V. ACKNOWLEDGEMENTS

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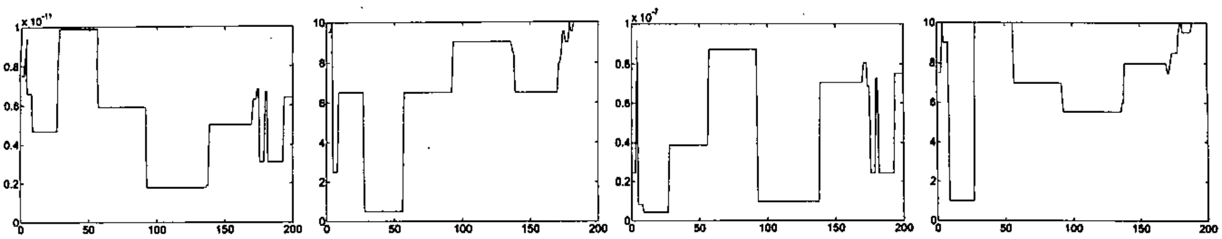
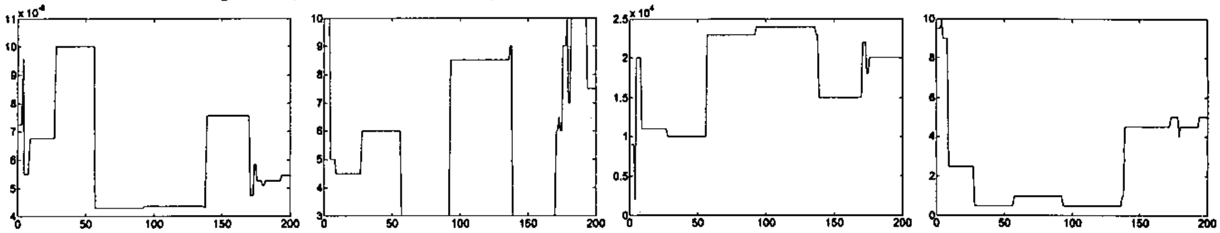
Table III: settings and results of the design process.

| | Definition of the search space | | Results of the design process | |
|---|---|---|---|---|
| | **Central value/step** | **Tolerance range/step** | **Final central value** | **Final tolerance** |
| | | **Case 1** | | |
| **C1** | [0.1pF,10pF]/0.1pF | [0.5%,10%]/0.5% | 6.4 pF | 10% |
| **C2** | [0.1nF,100nF]/0.1nF | [0.5%,10%]/0.5% | 75.2 nF | 10% |
| **C3** | [0.1nF,100nF]/0.1nF | [0.5%,10%]/0.5% | 54.4 nF | 7.5% |
| **R4** | [1kΩ,50kΩ]/1kΩ | [0.5%,10%]/0.5% | 20 kΩ | 5% |
| **R7** | [10Ω,5kΩ]/10Ω | [0.5%,10%]/0.5% | 420 Ω | 7.5% |
| | | **Case 2** | | |
| **C1** | [1pF,1nF]/1pF | [0.5%,50%]/0.5% | 4 pF | 50% |
| **C2** | [0.1nF,100nF]/0.1nF | [0.5%,50%]/0.5% | 99.2 nF | 50% |
| **C3** | [0.1nF,100nF]/0.1nF | [0.5%,50%]/0.5% | 98.6 nF | 27% |
| **R4** | [10kΩ,100kΩ]/1kΩ | [0.5%,50%]/0.5% | 10 kΩ | 31.5% |
| **R7** | [100Ω,10kΩ]/10Ω | [0.5%,50%]/0.5% | 100 Ω | 29% |



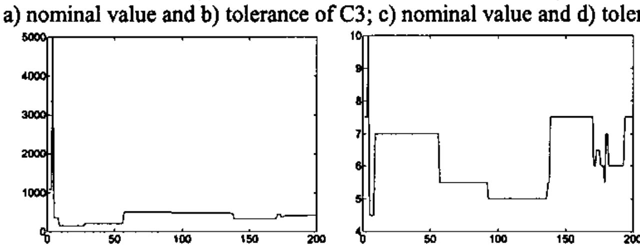
a) b) c) d)

Figure 2 a) nominal value and b) tolerance of C1; c) nominal value and d) tolerance of C2



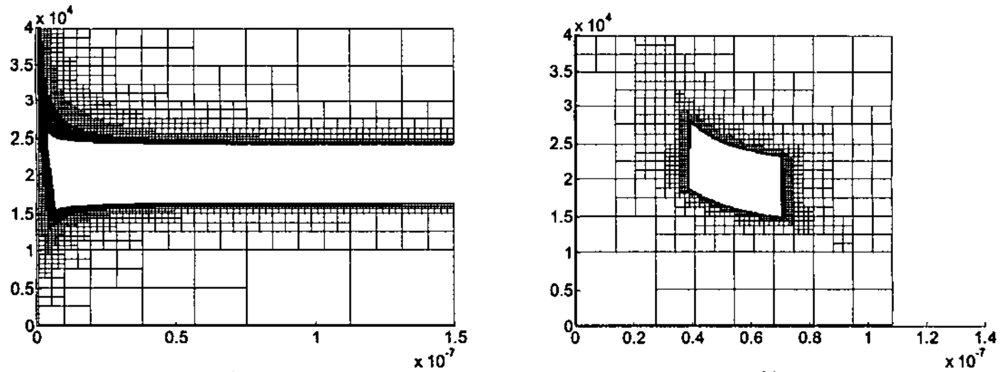
a) b) c) d)

Figure 3 a) nominal value and b) tolerance of C3; c) nominal value and d) tolerance of R4



a) b)

Figure 4. a) nominal value and b) tolerance of R7.




a) b)

Figure 5. ROA boundaries reconstruction. Projections in the planes of a) [$C_2$,$R_4$] (horizontal axis = $C_2$, vertical axis = $R_4$) and b) [$C_3$,$R_4$] (horizontal axis = $C_3$, vertical axis = $R_4$).