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Using interval arithmetic for robust state feedback design

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Abstract

A problem of a modal P-regulator synthesis for a linear multivariable dynamical system with uncertain (interval) parameters in state space is considered. The designed regulator has to place all coefficients of the system characteristic polynomial within assigned intervals. We have developed the approach proposed earlier in Dugarova and Smagina (Avtomat. i Telemech. 11 (1990) 176) and proved a direct correlation between interval system controllability and existence of robust modal P-regulator. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

One of the important tasks of feedback design is to provide robustness of closed-loop control systems when a plant includes parametrical uncertainties and/or is affected by disturbances. If the uncertainties can be described by intervals with known lower and upper bounds then the plant can be presented as interval dynamical system. Numerous methods of robust regulator synthesis created after appearance of Kharitonov Theorem use interval transfer function to model the system dynamics. As a result their application can be difficult for a wide class of the dynamical objects that need to be approximated by state space models because of their physical structure (for instance, an airplane motion or a chemical process). The necessity to develop an interval approach to robust regulator design for a plant with interval parameters in the state space has resulted in the studies [4,5,13,14] initiated by the authors. The methods discussed in [17,18] offer a solution for this problem without using classic interval arithmetic [10,8]. Moreover, application of the approach from [17,18] is limited to the systems with dynamical and input matrices of special structure. For interval systems of general type a new methodology discussed in the previous works of the authors [4,5,13,14] has been developed. The new approach based on the properties of non-standard interval arithmetic [9] and the algorithms from [2] provides simple formulae for regulator synthesis. Further development of the techniques introduced earlier in [5] has resulted in a new proof of the problem solvability conditions and a modified method of the robust P-regulator design presented in the paper. Note that an algorithm proposed below and the technique described in [12] can be used to obtain stabilizing regulators with real and, in some cases, interval gain coefficients. The

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revised solvability conditions show a direct correlation between interval system controllability and existence of a modal P-regulator. This paper can be considered as development of the ideas discussed earlier in [15,16].

2. Problem statement

Let us consider a linear multivariable control system in the state space

$$\dot{x} = [A]x + [B]u, \tag{1}$$

where $x = x(t)$ is a state n vector and $u = u(t)$ is an input r vector. The elements $[a_{ij}]$, $[b_{ik}]$, $i,\ j = 1,\dots,n$, $k = 1,\dots,r$ of $n \times n$ matrix $[A]$ and $n \times r$ matrix $[B]$ are intervals (interval numbers) with known upper and lower bounds [10]. These matrices describe the sets of matrices $A \in [A]$, $B \in [B]$ with real elements $a_{ij} \in [a_{ij}]$, $b_{ik} \in [b_{ik}]$.[1] Note that all interval numbers considered in this article are regular intervals [1].

For the robust state feedback control

$$u = Kx, \tag{2}$$

a modal P-regulator problem is to find a real $r \times n$ matrix K satisfying the inclusions

$$\det(sI - A - BK) \subseteq [D(s)] \tag{3}$$

for every real matrix $A \in [A]$, $B \in [B]$ where $[D(s)]$ is an assigned interval asymptotically stable polynomial

$$[D(s)] = s^n + [d_{n-1}]s^{n-1} + \cdots + [d_1]s + [d_0] \tag{4}$$

that describes a set of asymptotically stable polynomials $D(s) = s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0$ with real coefficients $d_i \in [d_i]$. The methodology discussed in [6] can be recommended for studies and selection of asymptotically stable interval polynomials.

3. Main result

Case $r = 1$: If $r = 1$ then $[B] = [b]$ is a column vector, $K = k$ is a row vector. The following criterion based on the result presented in [7] has been introduced in [5].

Controllability criterion 1 (*Sufficient*). The pair ($[A]$, $[b]$) is controllable for any $A \in [A]$, $b \in [b]$ if an interval controllability $n \times n$ matrix

$$[Y] = ([b], [A] * [b], \dots, [A]^{n-1} * [b]) \tag{5}$$

satisfies the condition

$$0 \notin \mathrm{Det}[Y]. \tag{6}$$

In (5) and (6) interval multiplication "$*$" for intervals $[a] = [a_l, a_s]$ and $[b] = [b_l, b_s]$ is defined as $[a] * [b] = \{\min(a_l b_l, a_l b_s, a_s b_l, a_s b_s),\ \max(a_l b_l, a_l b_s, a_s b_l, a_s b_s)\}$ [10] and Det $[\bullet]$ denotes an interval extension [10] of the function $\det(\bullet)$.

[1] A real number can be referred to as a degenerate [10] or a point [1] interval.

The above criterion differs from the one presented in [17] in its applicability to systems with interval matrices of general structure although in some cases it might require additional computational work.

Introduce an interval $n \times n$ matrix

$$[P] = [Y] * \begin{bmatrix} -[\alpha_1] & -[\alpha_2] & \cdots & -[\alpha_{n-1}] & -1 \\ -[\alpha_2] & -[\alpha_3] & \cdots & -1 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ -1 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{7}$$

and an interval row n vector

$$[f] = ([d_0]\Theta[\alpha_0], [d_1]\Theta[\alpha_1], \ldots, [d_{n-1}]\Theta[\alpha_{n-1}]), \tag{8}$$

where $[\alpha_{n-1}], \ldots, [\alpha_0]$ are the coefficients of the interval matrix $[A]$ characteristic polynomial $[\phi(s)] = s^n + [\alpha_{n-1}]s^{n-1} + \cdots + [\alpha_1]s + [\alpha_0]$. Interval polynomial (8) contains the characteristic polynomials $\phi(s) = s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0$ of all real matrices $A \in [A]$. In (8) "Θ" denotes a non-standard interval subtraction defined as $[a]\Theta[b] = \{\min(a_l - b_l, a_s - b_s),\ \max(a_l - b_l, a_s - b_s)\}$ [9].

Let us define a width of an interval number as $w[a] = w[a_l, a_s] = (a_s - a_l)$ [10].

Theorem 1. *If the pair* $([A],[b])$ *is controllable and the widths of the polynomial coefficients satisfy the inequalities* $w[d_i] > w[\alpha_i]$, $i = 0, \ldots, n-1$ *then a row n vector k of a stabilizing state feedback might be calculated from the interval inclusion*

$$k * [P] \subset [f]. \tag{9}$$

The proof of Theorem 1 is shown in the appendix.

The methods described in [12] can be recommended to obtain a real vector k or a set of all real vectors $\{k\}$ satisfying condition (9). In general, according to [12], to find the set $\{k\}$ can be a complex computational task. To simplify the calculations let us solve (9) for a real row vector k using the approach developed in [2].

Theorem 2. *Vector inclusion* (9) *has a solution of the form* [2]

$$k = M[f]\tilde{P}^{-1} \tag{10}$$

if the pair $([A],[b])$ *is controllable and for assigned asymptotically stable interval polynomial* (4) *the following inclusion*

$$M[f]\tilde{P}^{-1} * [P] \subset [f] \tag{11}$$

takes place.

In (10) and (11) matrix $\tilde{P} \in [P]$ is a real nonsingular matrix, $M[\bullet]$ denotes a real matrix (vector) of interval elements midpoints [3] [10].

Corollary. *The pair* $([A],[b])$ *controllability is a necessary condition for the modal P-regulator* $u = kx$ *existence*:

Remark 1. If a regular interval number $[a]$ is not a degenerate (point) interval then it has a positive width $w[a] = w[a_l, a_s] = (a_s - a_l) > 0$. So, the elements of interval vector $[f]$ (8) have to have positive widths and,

[2] For system with real (noninterval parameters) formula (9) coincides with the well-known Ackerman's formula.

[3] A midpoint for an interval number $[a]$ is defined as $m[a] = m[a_l, a_s] = (a_s - a_l)/2$.

therefore, they must satisfy the inequalities $w[d_i] > w[\alpha_i]$, $i=0,\dots,n-1$. Thus, it is recommended to select an asymptotically stable or a stable interval polynomial $[D(s)]$ (4) with the interval coefficients $[d_i], i=0,\dots,n-1$ that should be wide enough to guarantee the required inequalities.

In practice, a real matrix $\tilde{P}$ may be chosen as $\tilde{P}=M[P]$. Let us obtain restrictions on the interval coefficient widths of the assigned stable polynomial $[D(s)]$ to ensure the problem solvability.

Theorem 3. *If the pair* $([A],[b])$ *is controllable according the* Controllability criterion 1 *and* $\tilde{P}=M[P]$ *then inclusion* (9) *has a solution of the form*

$$k = M[f]M[P]^{-1}, \tag{12}$$

provided that the widths $w[d_j]$, $j=0,\dots,n-1$ *of the interval polynomial* $[D(s)]$ *coefficients satisfy the inequalities*

$$\sum_{i=1}^{n} abs(k_i)w[p_{ij}] + w[\alpha_j] < w[d_j], \tag{13}$$

where k_i *are the row vector* k *elements*, $w[p_{ij}]$ *are the widths of the elements of matrix* $[P]$ (7) [16].

Remark 2. For some control systems it is possible to calculate vector k (12) that does not satisfy some of inequalities (13). In order to comply with restrictions (13) we can try to increase the intervals $[d_j]$, $j=0,\dots,n-1$ widths without losing the interval polynomial $[D(s)]$ stability and then recalculate vector k (12).

Therefore, a stabilizing (non-modal) state feedback control $u=kx$ exists if the pair $([A],[b])$ is controllable and the coefficients of some (used to be assigned) stable nterval polynomial $[D(s)]$ (4) satisfy inequalities (13).

Case $r \geqslant 2$: For multivariable interval control system we use a concept of interval matrix rank introduced in [14].

For an interval $l \times p$ matrix $[C]$ we consider all minors $\det[C]_{m\times m}$ of the order $m \leqslant \min(l,p)$ where $\det[\bullet]$ denotes a united extension [10] of the function det. These minors are equal to some interval numbers. A minor is referred to as a singular if it contains zero.

Definition. An interval matrix $[C]$ rank denoted as $\operatorname{rank}[C]$ is equal to the maximal order of its nonsingular minors.

Based on the above definition for $n \times n$ matrix $[A]$ and $n \times r$ matrix $[B]$ we introduce

Controllability criterion 2. The pair $([A],[B])$ is controllable for any $A \in [A]$, $B \in [B]$ if and only if the interval controllability matrix

$$[Y] = ([B],[A]*[B],\dots,[A]^{n-1}*[B]) \tag{14}$$

satisfies the equation

$$\operatorname{rank}[Y] = n. \tag{15}$$

For a controllable system condition (15) means that selecting different columns of matrix $[Y]$ (14) we always can form an $n \times n$ minor $\det[Y]_{n\times n}$ such that $0 \notin \det[Y]_{n\times n}$.

If we replace the minor $\det[Y]_{n\times n}$ with an interval extension $\operatorname{Det}[Y]_{n\times n}$ then we can consider the following computational

Controllability criterion 3 (*Sufficient*). The pair $([A],[B])$ is controllable for any $A \in [A]$, $B \in [B]$ if

$$0 \notin \operatorname{Det}[Y]_{n\times n}. \tag{16}$$

Assume that for any $A \in [A]$ and $B \in [B]$ the pair $([A], [B])$ is controllable and cyclic [11]. Then we can "almost always" find a real r vector q to make the pair $([A], [B] * q)$ controllable. Denoting $[b] = [B] * q$ and applying Theorem 1 we can calculate real row vector k (10) and use the formula

$$K = qk \tag{17}$$

to compute a real gain $r \times n$ matrix K for feedback control (2). If the cyclic condition is violated for some $A \in [A]$ and $B \in [B]$ then according to [19] for the controllable pair (A, B) a feedback matrix $\tilde{K}$ "almost always" can be found to make the pair $(A + B\tilde{K}, B)$ is cyclic.

Thus, the interval pair $([A], [B])$ controllability is a necessary condition of the modal P-regulator existence.

Consider the following algorithm for synthesis of stabilizing P-regulator (2).

Step 1: For the pair $([A], [B])$ analyze controllability criterion (16) to determine if the problem has a solution. Note that for the pair $([A], [b])$ conditions (5) and (6) can be checked out.

Step 2: Calculate the coefficients $[\alpha_i]$ of the matrix $[A]$ characteristic polynomial or intervals containing them [3].

Step 3: Select coefficients $[d_i], i=0,\dots,n-1$ of interval polynomial (4) such that the inequalities $w[d_i] > w[\alpha_i]$ take place for all $i = 0,\dots,n-1$ and $[D(s)]$ is (asymptotically) stable. If for some i both these conditions cannot be satisfied then the problem has no solution.

Step 4: If $r = 1$ then go to Step 5, otherwise choose a real r vector v to make the pair $([A], [B] * v)$ controllable.

Step 5: Compute $k = M[f]\tilde{P}^{-1}$ or $k = M[f]M[P]^{-1}$ if matrix $\tilde{P} = M[P]$. If inequalities (13) hold then go to Step 6, otherwise return to the Step 3 and increase the widths of the intervals $[d_i], i = 0,\dots,n-1$ preserving, if possible, (asymptotic) stability of polynomial $[D(s)]$ (4).

Step 6: If $r = 1$ then $K = k$. If $r \geqslant 2$ then $K = qk$.

Remark 3. Computer implementation of the proposed algorithm depends on availability of special software to perform interval arithmetic operations and efficiency of algorithms needed to calculate expressions involving interval matrices. The algebraic methods based on the formulae with the least possible number of interval multiplications are recommended for reducing interval results widths. Note that computational complexity of the proposed method is also related to the number of interval parameters in the model and the algorithm performance time can increase for the models of high orders and with large number of interval parameters. It might result in consumption of additional computing resources but it should not be an issue for modern powerful computers. For low-order control systems with a fewer number of interval parameters the time needed to run the developed algorithm on the computer does not present a problem. For instance, it does not take more than a several minutes of computing time to perform all calculations in the example shown below [3].

4. Example

Let us consider a stabilization problem for the helicopter longitudinal motion speed model [7] described by a linear differential equation of form (1) with $n = 3$, $r = 2$ and interval matrices $[A]$ and $[B]$

$$[A] = \begin{bmatrix} [a_{11}] & [a_{12}] & [a_{13}] \\ [a_{21}] & [a_{22}] & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad [B] = \begin{bmatrix} [b_{11}] & 0 \\ 0 & [b_{22}] \\ 0 & 0 \end{bmatrix}, \tag{18}$$

where

$$[a_{11}] = [-0.031, -0.0128], \qquad [a_{12}] = [-3.4, -0.1], \qquad [a_{21}] = [-0.00077, -0.0007],$$

$$[a_{22}] = [-0.32, -0.31], \qquad [a_{13}] = [-9.8, -9.8],$$
$$[b_{11}] = [-18, -15], \qquad [b_{22}] = [-3.3, -3]. \tag{19}$$

In the state vector $x = (x_1, x_2, x_3)^{\tau}$: x_1 is the deviation of the longitudinal motion projection, x_2 is the angular speed deviation, x_3 is the pitch angle deviation.

The task is to find a real 2×3 matrix K such that for every real matrix $A \in [A]$ and $B \in [B]$ the characteristic polynomial coefficients of the closed-loop matrix $A + BK$ are located within the interval coefficients of the given asymptotically stable interval polynomial

$$[D(s)] = s^3 + [3,4]s^2 + [2,8]s + [0.5, 5.5]. \tag{20}$$

It can be shown that the pair $([A],[B])$ is controllable and for the selected vector $q = (0.8, 1.2)^{\tau}$ the pair $([A],[b])$ is also controllable [3]. Calculations of the coefficients of the matrix $[A]$ characteristic polynomial $[\phi(s)] = \det(sI - [A]) = s^3 + [\alpha_2]s^2 + [\alpha_1]s + [\alpha_0]$ give the intervals $[\alpha_2] = [0.3228, 0.351], [\alpha_1] = [0.00135, 0.00985], [\alpha_0] = [-0.00755, -0.00686]$ that can be used to compute interval matrix (7) as follows:

$$[P] = \begin{bmatrix} [-43.3833, -30.5269] & [-10.036, 4.5404] & [12, 14.4] \\ [-0.0920, 0.0953] & [-0.1161, 0.2556] & [3.6, 3.96] \\ [-0.1161, 0.2556] & [3.6, 3.96] & 0 \end{bmatrix}.$$

Since $\det M[P] \neq 0$ and $w[d_i] > w[\alpha_i], i=0,1,2$ then $k = M[f](M[P])^{-1} = (-0.0793, 1.1160, 1.2471)$. According to [3] inequalities (13) are satisfied for this vector k so it does not require additional tuning described in Step 5 of the above algorithm. The gain matrix of the modal P-regulator has the following form [3]:

$$K = qk = \begin{bmatrix} -0.0634 & 0.8228 & 0.9977 \\ -0.0958 & 1.3392 & 1.4963 \end{bmatrix}.$$

Simulation of the closed-loop system responses for different parameter values from bounds (19) shows asymptotic stability of the closed-loop control systems (see Fig. 1).

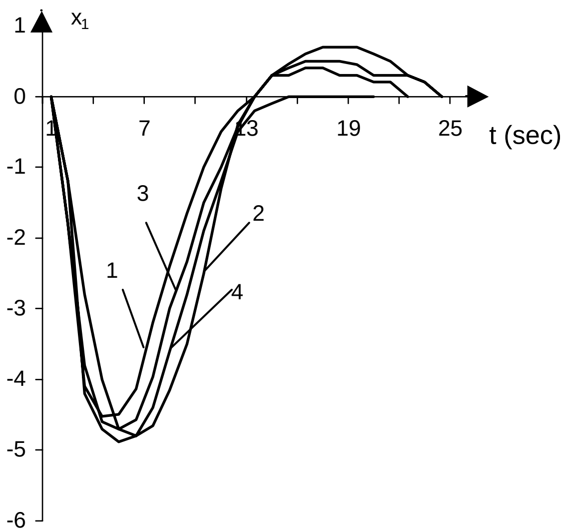

Fig. 1. State responses for different systems (1–4) with parameters from intervals (19).

5. Conclusion

New aspects of the robust regulator design method for a plant with uncertain parameters have been considered. The developed approach can be applied for a dynamical system with interval matrices of general structure provided that the united or interval extensions of interval matrix determinant and coefficients of interval matrix characteristic polynomial can be obtained. It has been shown that a problem of ensuring an interval control system (asymptotic) stability can be solved by designing a real modal robust regulator that places all characteristic polynomial coefficients of the closed-loop system within the interval coefficients of an assigned (asymptotically) stable interval polynomial. Necessary and sufficient conditions of the problem solution have been discussed and an algorithm for calculations of real gain feedback coefficients has been proposed in the paper. The developed approach can be applied for obtaining a set of robust regulators for an interval dynamical plant by using advanced techniques to find an interval linear algebraic system solution.

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Appendix

Proof of the Theorem 1. If the pair $([A],[b])$ is controllable then all pairs (A,b) where $A\in[A], b\in[b]$ are controllable. According to [11] for a controllable pair (A,b) a feedback row vector k can be calculated from the following matrix equation:

$$k(b,Ab,\dots,(A)^{n-1})\begin{bmatrix} -\alpha_1 & -\alpha_2 & \cdots & -\alpha_{n-1} & -1 \\ -\alpha_2 & -\alpha_3 & \cdots & -1 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ -1 & 0 & \cdots & 0 & 0 \end{bmatrix} + (\alpha_0,\alpha_1,\dots,\alpha_{n-1}) = (d_0,d_1,\dots,d_{n-1}), \tag{A.1}$$

where $\alpha_0,\alpha_1,\dots,\alpha_{n-1}$ are the matrix A characteristic polynomial coefficients, $d_0,d_1,\dots,d_{n-1}$ are the coefficients of the assigned asymptotically stable polynomial $D(s)=s^n+d_{n-1}s^{n-1}+\cdots+d_1s+d_0$ such as $d_i\in[d_i], i=0,\dots,n-1$.

The characteristic polynomial coefficients $\alpha_i(A), i=0,\dots,n-1$ can be considered as multi-linear functions of the matrix A elements. Denote the row vector from the left-hand side of (A.1) by $f(k,A,b)$. Its coordinates are rational functions of the matrices k,A,b elements. Selecting an interval extension $F(k,[A],[b])$ of the function $f(k,A,b)$ when $A\in[A], b\in[b]$ we can represent the left-hand side of Eq. (A.1) in the interval form

$$F(k,[A],[b]) = k*[P] + ([\alpha_0],[\alpha_1],\dots,[\alpha_{n-1}]), \tag{A.2}$$

where the interval $n\times n$ matrix $[P]$ is defined in (7), $[\alpha_i], i=0,\dots,n-1$ are interval extensions of the rational functions $\alpha_i(A), A\in[A]$.

If we can find a real row vector k satisfying the following inclusion:

$$k*[P] + ([\alpha_0],[\alpha_1],\dots,[\alpha_{n-1}]) \subset ([d_0],[d_1],\dots,[d_{n-1}]) \tag{A.3}$$

then from Eq. (A.1) we can calculate $d_i, i=0,\dots,n-1$ for every $A \in [A]$ and $b \in [b]$ and, according to (A.3), they are to belong to the correspondent interval coefficients $d_i \in [d_i], i=0,\dots,n-1$. Function $F(k,[A],[b])$ (A.2) is a sum of an unknown interval row vector $k*[P]$ and a known interval row vector $([\alpha_0], [\alpha_1], \dots, [\alpha_{p-1}])$. In order to transform (A.3) into form (9) we would subtract the known interval vector from both sides of relation (A.3) leaving the unknown member $k*[P]$ in the left-hand side. Since the regular interval subtraction of an interval number $[a]$ from itself results in a nonzero interval $[a]-[a] \neq 0$[10] we can not use this interval arithmetic operation to solve Eq. (A.3) for $k*[P]$. The desired zero result can be achieved if we apply a nonstandard interval subtraction $[a]\Theta[a]=0$ introduced in [9]. Application of this arithmetic operation to Eq. (A.3) results in formula (9).

The inequalities $w[d_i] > w[\alpha_i], i=0,\dots,p-1$ follow from (8) and the definition of the regular interval width. Theorem 1 has been proved.

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