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STABILITY

Absolute Stability of an Interval Family of Nonlinear Dynamic Systems with Nonlinear Feedback

R. S. Ivlev

Institute for Problems of Informatics and Control, Center for Physicomathematical Studies, Ministry of Education and Science of Kazakhstan, ul. Pushkina 125, Almaty, 480100 Kazakhstan

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Abstract—Based on the direct Lyapunov method, the sufficient conditions of the absolute stability of an interval family of Lurie nonlinear dynamic systems are obtained. Checking of these conditions requires small computational costs.

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INTRODUCTION

Many applications come up against inaccuracies in the initial data. In the context of control theory, this problem is very topical. In particular, most of the problems of investigating features of dynamic systems under an interval of uncertainty of parameters remain to be unsolved.

In this paper, we consider a nonlinear dynamic system with a nonlinearity of the sector type [1, 2] and parameters given in the form of intervals [3]. Unfortunately, the number of publications devoted to this line of investigation is very small. Among them, paper [4] is worth noting, in which the authors consider a discrete case where a linear part is given as a family of polynomials. In [4], robust modifications of frequency criteria of absolute stability under uncertainty in the linear part of the system were obtained. In contrast to [4], in this paper, the system is considered in a state space. It is known that the problem of investigating the stability of a dynamic system in the state space even in a linear case is *NP*-hard [5]. In this connection, to investigate the absolute stability of a dynamic system with a nonlinearity of the sector type and interval parameters assigned in the state space, an approach that uses the direct Lyapunov method and requires small computational costs is proposed.

1. STATEMENT OF THE PROBLEM

Assume that a disturbed motion of a dynamic system is described in the state space in the form of the following differential equation with parameters that were given inaccurately:

$$\dot{x}(t) = (A_c + \Delta A)x(t) + (b_c + \Delta b)\varphi(\sigma), \quad x(t_0) = x_0, \quad t \in [t_0, \infty). \tag{1.1}$$

Here, t is an independent variable (time) and $x(t)$ is a state vector whose components are functions ($x_i(t)$ that are continuously differentiable on $[t_0, \infty)$; i.e., $x_i(t) \in C^1[t_0, \infty)$, $i = 1, 2, \ldots, n$ (at the initial instant of time t_0, the value of the state vector is supposed to be known, $x_0 \in R^n$). The fixed matrix $A_c \in R^{n \times n}$ and vector $b_c \in R^n$ are known. The parametric uncertainty in system (1.1) is represented by the unknown fixed matrix $\Delta A \in R^{n \times n}$ and vector $\Delta b \in R^n$; it is assumed that they can take values from the given interval matrix and vector with known boundaries

$$\Delta A \in [-\Delta_A, \Delta_A], \quad \Delta_b \in [-\Delta_b, \Delta_b],$$

where $\Delta_A = |\Delta_A| \in R^{n \times n}$ and $\Delta_b = |\Delta_b| \in R^n$ are a given fixed matrix and a vector. As applied to matrices and vectors, the operation of taking the absolute value $|\bullet|$ has a component-wise meaning. Also, it is assumed that $-A_c \notin [-\Delta_A, \Delta_A]$ and $-b_c \notin [-\Delta_b, \Delta_b]$. This condition implies that some entries of the interval matrix $[A_c - \Delta_A, A_c + \Delta_A]$ and the interval vector $[b_c - \Delta_b, b_c + \Delta_b]$ may not contain zero. The function $\varphi(\sigma)$ is continuously differentiable, $\varphi : R \longrightarrow R$ such that constraints of the sector type are valid (a function graph is located in the sector between the straight lines $\varphi = 0$ and $\varphi = \mu\sigma$, $\mu \in R$, $\mu > 0$). These constraints can be represented as follows:

$$0 \le \varphi/\sigma \le \mu, \tag{1.2}$$

where $\sigma \neq 0$, while, for $\sigma = 0$, it is necessary that the condition $\varphi(0) = 0$ be satisfied. The value of $\sigma \in R$ is found by the expression

$$\sigma = r^T x,$$

where $r \in R^n$ is an assigned numerical n-by-1 vector.

Double inequality (1.2) can be rewritten in the form of a single inequality. Taking into account the expression for σ, we have

$$(\mu\sigma - \varphi)\varphi = \mu\varphi r^T x - \varphi^2 = F(x, \varphi) \geq 0. \qquad (1.3)$$

The left-hand side of inequality (1.3) is a quadratic form of the variables x and φ, which describes a nonlinear constraint on x and φ.

For any combinations of fixed values of the matrix $\Delta A \in [-\Delta_A, \Delta_A]$ and the vector $\Delta b \in [-\Delta_b, \Delta_b]$, differential equation (1.1) satisfies the existence and uniqueness conditions for any initial condition $x_0 \in R^n$. In view of the properties of the function φ, for the initial condition $x_0 = 0$, we have the trivial solution $x(t, t_0, x_0) = x(t, t_0, 0) = x(t) \equiv 0$, which is an equilibrium position of system (1.1).

The problem consists in finding conditions, under which the equilibrium position $x(t) \equiv 0$ of system (1.1) is absolutely stable with various combinations of values of the matrix ΔA from the given interval matrix $[-\Delta_A, \Delta_A]$ and the vector Δb from the given interval vector $[-\Delta_b, \Delta_b]$.

2. BASIC RESULT

Assume that a pair of interval matrices $([A_c - \Delta_A, A_c + \Delta_A], [b_c - \Delta_b, b_c + \Delta_b])$ is stabilized; i.e., for any $\Delta A \in [-\Delta_A, \Delta_A]$ and $\Delta_b \in [-\Delta_b, \Delta_b]$, the pair $(A_c + \Delta A, b_c + \Delta b)$ is stabilized.

The stated problem is solved using the direct Lyapunov method by choosing a Lyapunov function as a quadratic form

$$V(x) = x^T H x. \qquad (2.1)$$

Here, $H \in R^{n \times n}$, $H = H^T \triangleright 0$ is a symmetric positive definite matrix determined from the Lyapunov equation

$$A_c^T H + H A_c + I = -G_{11}, \qquad (2.2)$$

where $G_{11} \in R^{n \times n}$, $G_{11} = G_{11}^T \triangleright 0$ is a symmetric positive definite matrix, and I is the identity nth matrix of order n.

In what follows, we shall use the quadratic equation

$$\alpha\tau^2 + \beta\tau + \gamma = 0 \qquad (2.3)$$

for $\tau \in R$, where

$$\alpha = \left(\frac{\mu}{2}\right)^2 r^T r,$$

$$\beta = \mu r^T (H b_c + G_{12}) - 1,$$

$$\gamma = b_c^T H H b_c + G_{12}^T (2 H b_c + G_{12}) + G_{22},$$

while $G_{12} \in R^n$ and $G_{22} \in R$ are some n-by-1 vector and scalar such that the matrix $G \in R^{(n+1) \times (n+1)}$, which can be represented in the block form,

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{12}^T & G_{22} \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix},$$

is symmetric positive definite.

We generate the block matrices D_c, Δ_D, $\tilde{G} \in R^{(n+1) \times (n+1)}$

$$D_c = \begin{pmatrix} A_c^T H + H A_c & \frac{\tau\mu}{2} r + H b_c \\ \frac{\tau\mu}{2} r^T + b_c^T H & -\tau \end{pmatrix},$$

$$\Delta_D = \begin{pmatrix} \Delta_A^T |H| + |H| \Delta_A & |H| \Delta_b \\ \Delta_b^T |H| & 0 \end{pmatrix},$$

$$\tilde{G} = \begin{pmatrix} G_{11} + I & G_{12} + h \\ G_{21} + h^T & G_{22} + h^T h \end{pmatrix},$$

where

$$h = -\left(H b_c + \frac{\tau\mu}{2} r + G_{12}\right). \qquad (2.4)$$

Introduce the set of vectors $z \in R^{n+1}$

$$Z = \{z \in R^{n+1} | z = (z_i), z_i \in \{-1, 1\}, \\ i = 1, 2, \ldots, n+1\}. \qquad (2.5)$$

Using (2.5), we introduce the matrices $\tilde{G}_z \in R^{(n+1) \times (n+1)}$ described for each $z \in Z$ as follows:

$$(\tilde{G}_z)_{ij} = \begin{cases} -\tilde{G}_{ij} + (\Delta_D)_{ij}, & \text{if } z_i z_j = +1 \\ -\tilde{G}_{ij} - (\Delta_D)_{ij}, & \text{if } z_i z_j = -1. \end{cases}$$

Introduce the notation $\rho(\bullet)$ and $\lambda(\bullet)$ for the spectral radius and the minimum eigenvalue of a real quadratic symmetric matrix, respectively.

The absolute-stability conditions of the investigated system are given by the following theorem.

Theorem. Assume that, for the assigned matrices $A_c, \Delta_A \in R^{n \times n}$, vectors $b_c, \Delta_b, r \in R^n$, positive scalar $\mu \in R$, and a symmetric positive definite matrix $G \in R^{(n+1) \times (n+1)}$, the following conditions are satisfied:

(1) Lyapunov equation (2.2) has a symmetric positive definite solution $H \in R^{n \times n}$, $H = H^T \triangleright 0$;

(2) quadratic equation (2.3) has a real nonnegative root;

(3) one of the following conditions is satisfied: (a) $\rho(\Delta_D) < \lambda(\tilde{G})$ and (b) the matrices $\tilde{G}_z$ for any $z \in Z$ are negative definite.

Then, the equilibrium position $x(t) \equiv 0$ of investigated system (1.1) is absolutely stable for a chosen class of nonlinearities.

Proof. Introduce the notation $A = A_c + \Delta_A$ and $b = b_c + \Delta_b$ for $\Delta A \in [-\Delta_A, \Delta_A]$ and $\Delta b \in [-\Delta_b, \Delta_b]$ Compute the first-order derivative in time of function (2.1) in the motion trajectories of investigated system (1.1)

$$\dot{V}(x)|_{(1.1)} = \dot{x}^T Hx + x^T H\dot{x}$$
$$= (Ax + b\varphi)^T Hx + x^T H(Ax + b\varphi).$$

Show that when the conditions of the theorem are satisfied, the found derivative is negative on motion trajectories of the investigated system in the part of the R^n-by-R space; this part is formed by constraints of sector type (1.3). For this purpose, we use the S-procedure [6], which makes it possible to obtain the S-form for $\tau \ge 0$

$$S(x, \varphi) = \dot{V}(x) + \tau F(x, \varphi) = x^T(A^T H + HA)x$$
$$+\left(\frac{\tau\mu}{2}r^T + b^T H\right)x\varphi + x^T\left(\frac{\tau\mu}{2}r + Hb\right)\varphi - \tau\varphi^2$$
$$= x^T(A_c^T H + HA_c)x + \left(\frac{\tau\mu}{2}r^T + b_c^T H\right)x\varphi$$
$$+ x^T\left(\frac{\tau\mu}{2}r + Hb_c\right)\varphi - \tau\varphi^2$$
$$+ x^T(\Delta A^T H + H\Delta A)x + \varphi\Delta b^T Hx + x^T H\Delta b\varphi.$$

We make simple transforms

$$\begin{pmatrix} x^T & \varphi \end{pmatrix}\begin{pmatrix} \Delta A^T H + H\Delta A & H\Delta b \\ \Delta b^T H & 0 \end{pmatrix}\begin{pmatrix} x \\ \varphi \end{pmatrix}$$
$$\le \left|\begin{pmatrix} x^T & \varphi \end{pmatrix}\begin{pmatrix} \Delta A^T H + H\Delta A & H\Delta b \\ \Delta b^T H & 0 \end{pmatrix}\begin{pmatrix} x \\ \varphi \end{pmatrix}\right|$$
$$\le \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix}\begin{pmatrix} |\Delta A^T H + H\Delta A| & |H\Delta b| \\ |\Delta b^T H| & 0 \end{pmatrix}\begin{pmatrix} |x| \\ |\varphi| \end{pmatrix}$$
$$\le \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix}\begin{pmatrix} |\Delta A^T||H| + |H||\Delta A| & |H||\Delta b| \\ |\Delta b^T||H| & 0 \end{pmatrix}\begin{pmatrix} |x| \\ |\varphi| \end{pmatrix}$$
$$\le \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix}\begin{pmatrix} \Delta_A^T|H| + |H|\Delta_A & |H|\Delta_b \\ \Delta_b^T|H| & 0 \end{pmatrix}\begin{pmatrix} |x| \\ |\varphi| \end{pmatrix},$$

and estimate of the obtained S-form from above:

$$S(x, \varphi) \le \begin{pmatrix} x^T & \varphi \end{pmatrix} D_c \begin{pmatrix} x \\ \varphi \end{pmatrix} + \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix}\Delta_D\begin{pmatrix} |x| \\ |\varphi| \end{pmatrix}.$$

The first term in the right-hand side is a quadratic form of the variables x and φ. Let us show that this quadratic form is negative definite. From (2.3), we have

$$\alpha\tau^2 + \beta\tau + \gamma = \left(\frac{\mu}{2}\right)^2 r^T r\tau^2 + \mu r^T(Hb_c + G_{12})\tau - \tau$$
$$+ b_c^T HHb_c + G_{12}^T(2Hb_c + G_{12}) + G_{22} = -\tau \qquad (2.6)$$
$$+\left(Hb_c + \frac{\tau\mu}{2}r + G_{12}\right)^T\left(Hb_c + \frac{\tau\mu}{2}r + G_{12}\right) + G_{22} = 0.$$

Taking into account (2.4), we reduce equality (2.6) to the form

$$-\tau + h^T h + G_{22} = 0. \qquad (2.7)$$

Using (2.4), (2.7), and Lyapunov equation (2.2), we obtain

$$\begin{pmatrix} x^T & \varphi \end{pmatrix} D_c \begin{pmatrix} x \\ \varphi \end{pmatrix} = x^T(A_c^T H + HA_c)x$$
$$+\left(\frac{\tau\mu}{2}r^T + b_c^T H\right)x\varphi + x^T\left(\frac{\tau\mu}{2}r + Hb_c\right)\varphi - \tau\varphi^2$$
$$= x^T(A_c^T H + HA_c + I)x - x^T x - (h^T + G_{12}^T)x\varphi$$
$$- x^T(h + G_{12})\varphi - (G_{22} + h^T h)\varphi^2$$
$$= -x^T G_{11}x - x^T x - (h^T + G_{12}^T)x\varphi$$
$$- x^T(h + G_{12})\varphi - (G_{22} + h^T h)\varphi^2$$
$$= -\begin{pmatrix} x^T & \varphi \end{pmatrix}\begin{pmatrix} G_{11} + I & G_{12} + h \\ G_{12}^T + h^T & G_{22} + h^T h \end{pmatrix}\begin{pmatrix} x \\ \varphi \end{pmatrix}$$
$$= -(x + h\varphi)^T(x + h\varphi) - \begin{pmatrix} x^T & \varphi \end{pmatrix} G \begin{pmatrix} x \\ \varphi \end{pmatrix} < 0,$$
$$|x| \ne 0, \quad |\varphi| \ne 0.$$

Now, we show that, when one of the conditions (3a) or (3b) of the theorem is satisfied, the S-form is a negative definite quadratic form for any $\Delta A \in [-\Delta_A, \Delta_A]$ and

$\Delta b \in [-\Delta_b, \Delta_b]$. Assume that condition (3a) is satisfied; then, we have

$$S(x, \varphi) \le \begin{pmatrix} x^T & \varphi \end{pmatrix} D_c \begin{pmatrix} x \\ \varphi \end{pmatrix} + \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix} \Delta_D \begin{pmatrix} |x| \\ |\varphi| \end{pmatrix}$$

$$= -\begin{pmatrix} x^T & \varphi \end{pmatrix} \tilde{G} \begin{pmatrix} x \\ \varphi \end{pmatrix} + \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix} \Delta_D \begin{pmatrix} |x| \\ |\varphi| \end{pmatrix}$$

$$\le -\lambda(\tilde{G}) \begin{pmatrix} x^T & \varphi \end{pmatrix} \begin{pmatrix} x \\ \varphi \end{pmatrix} + \rho(\Delta_D) \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix} \begin{pmatrix} |x| \\ |\varphi| \end{pmatrix} < 0,$$

$$|x| \ne 0, \quad |\varphi| \ne 0.$$

Suppose that condition (3b) of the theorem is satisfied; then, we have

$$S(x, \varphi) \le \begin{pmatrix} x^T & \varphi \end{pmatrix} D_c \begin{pmatrix} x \\ \varphi \end{pmatrix} + \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix} \Delta_D \begin{pmatrix} |x| \\ |\varphi| \end{pmatrix}$$

$$= \begin{pmatrix} x^T & \varphi \end{pmatrix} (-\tilde{G}) \begin{pmatrix} x \\ \varphi \end{pmatrix} + \begin{pmatrix} |x^T| & |\varphi| \end{pmatrix} \Delta_D \begin{pmatrix} |x| \\ |\varphi| \end{pmatrix}$$

$$= \begin{pmatrix} x^T & \varphi \end{pmatrix} \tilde{G}_z \begin{pmatrix} x \\ \varphi \end{pmatrix}, \quad z \in Z.$$

By the condition of the theorem, we have

$$\begin{pmatrix} x^T & \varphi \end{pmatrix} \tilde{G}_z \begin{pmatrix} x \\ \varphi \end{pmatrix} < 0, \quad |x| \ne 0, \quad |\varphi| \ne 0, \ \forall z \in Z.$$

Hence, the S-form is negative definite for any $\Delta A \in [-\Delta_A, \Delta_A]$ and $\Delta b \in [-\Delta_b, \Delta_b]$. Taking into account condition (2) of the theorem, the first-order derivative in time of Lyapunov function (2.1) is negative on motion trajectories of the investigated system for any $\Delta A \in [-\Delta_A, \Delta_A]$ and $\Delta b \in [-\Delta_b, \Delta_b]$ in the part of the R^n-by-R space; here, this part is formed by constraints of sector type (1.3). Then, the equilibrium position $x(t) \equiv 0$ of the investigated system is absolutely stable for any values of $\Delta A \in [-\Delta_A, \Delta_A]$ and $\Delta b \in [-\Delta_b, \Delta_b]$. The theorem is proved.

3. NUMERICAL EXAMPLE

Consider system (1.1) for $n = 3$ and for the following numerical data:

$$A_c = \begin{pmatrix} -3.5 & 2.1 & -1.5 \\ 1.0 & -3.5 & 2.1 \\ 0.0 & 1.1 & -1.8 \end{pmatrix}, \quad b_c = \begin{pmatrix} 0.2 \\ 1.0 \\ 0.6 \end{pmatrix},$$

$$\Delta_A = \begin{pmatrix} 0.5 & 0.1 & 0.5 \\ 0.5 & 0.5 & 0.1 \\ 0.0 & 0.1 & 0.2 \end{pmatrix}, \quad \Delta_b = \begin{pmatrix} 0.1 \\ 0.0 \\ 0.1 \end{pmatrix}, \quad \mu = 0.15.$$

We investigate the absolute stability of this system using the proved theorem. For the symmetric positive definite matrix

$$G = \begin{pmatrix} 1.0 & 0.0 & 0.0 & 0.5 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.5 & 0.0 & 0.0 & 3.0 \end{pmatrix}$$

we have

$$G_{11} = \begin{pmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{pmatrix}, \quad G_{12} = \begin{pmatrix} 0.5 \\ 0.0 \\ 0.0 \end{pmatrix}, \quad G_{22} = 3.0.$$

Here, the matrix G_{11} coincides with the identity third-order matrix $G_{11} = I$ and is a symmetric positive definite matrix. The solution of Lyapunov equation (2.2) for the given numerical matrices, up to the third decimal position, has the form

$$H = \begin{pmatrix} 0.337 & 0.181 & 0.045 \\ 0.181 & 0.509 & 0.364 \\ 0.045 & 0.364 & 0.943 \end{pmatrix}.$$

In this case, the matrix H is symmetric positive definite (its eigenvalues are $\lambda_1(H) = 0.176$, $\lambda_2(H) = 0.443$, and $\lambda_3(H) = 1.170$). Hence, the first condition of the theorem is satisfied. To check the second condition, we compute the coefficients

$$\alpha = 0.017, \quad \beta = -0.856, \quad \gamma = 3.464.$$

For these numerical coefficients, quadratic equation (2.3) has two real positive solutions, namely, $\tau_1 = 4.433$ and $\tau_2 = 46.299$. The second condition of the theorem is also satisfied. It remains to check one of conditions (3a) or (3b). We take $\tau = \tau_1 = 4.433$ and check condition (3a). For this purpose, we compute the matrices

$$\Delta_D = \begin{pmatrix} 0.518 & 0.473 & 0.400 & 0.038 \\ 0.473 & 0.618 & 0.495 & 0.055 \\ 0.400 & 0.495 & 0.495 & 0.099 \\ 0.038 & 0.055 & 0.099 & 0.000 \end{pmatrix},$$

$$\tilde{G} = \begin{pmatrix} 2.000 & 0.000 & 0.000 & -0.476 \\ 0.000 & 2.000 & 0.000 & -0.497 \\ 0.000 & 0.000 & 2.000 & -0.484 \\ -0.476 & -0.497 & -0.484 & 4.433 \end{pmatrix}.$$

The eigenvalues of the matrix Δ_D are $\lambda_1(\Delta_D) = -0.029$, $\lambda_2(\Delta_D) = 0.066$, $\lambda_3(\Delta_D) = 0.124$, and $\lambda_4(\Delta_D) = 1.470$. Analogously, for the matrix $\tilde{G}$, we have the following characteristic values:

$\lambda_1(\tilde{G}) = 1.737$, $\lambda_2(\tilde{G}) = 2.000$, $\lambda_3(\tilde{G}) = 2.000$, and $\lambda_4(\tilde{G}) = 4.696$. As can be seen from these results, $\rho(\Delta_D) = \lambda_4(\Delta_D) = 1.470 < \lambda(\tilde{G}) = \lambda_1(\tilde{G}) = 1.737$; i.e., condition (3a) of the theorem is satisfied. Thus, all conditions of the theorem are satisfied, and, on the basis of the statement of the theorem, we conclude that the investigated system is absolutely stable.

CONCLUSIONS

The proposed approach based on the direct Lyapunov method makes it possible to investigate the absolute stability of a dynamic system with a nonlinearity of the sector type and interval parameters given in the state space; note that this approach requires small computational costs. This approach is also applicable to investigating the stability of dynamic systems in the state space with interval parameters and nonlinearities of other types.

REFERENCES

1. A. I. Lur'e, *Certain Nonlinear Problems of Automatic Control Theory* (Gostekhizdat, Moscow, 1951) [in Russian].
2. V. M. Popov, *Hyper-stability of Automatic Systems* (Nauka, Moscow, 1970) [in Russian].
3. G. Alefeld, and J. Herzberger, *Introduction to Interval Computations* (Academic, New York, 1983; Mir, Moscow, 1987).
4. E. I. Dzhuri, K. Premaratne, and M. M. Ekanaiake, "Robust Absolute Stability of Discrete Systems," Avtom. Telemekh., No. 3 (1999).
5. V. Kreinovich, A. Lakeyev, J. Rohn, et al., *Computational Complexity and Feasibility of Data Processing and Interval Computations* (Kluwer, Dordrecht, 1997).
6. A. Kh. Gelig, G. A. Leonov, and V. A. Yakubovich, *Stability of Nonlinear Systems with Nonunique Equilibrium States* (Nauka, Moscow, 1978) [in Russian].