Computational Complexity Of Sentences Over Fields *

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Abstract

Hilbert's Irreducibility Theorem is applied to find the upper bounds of the time complexities of various decision problems in arithmetical sentences and the following results are proved:

1. The decision problem of $\forall\exists$ sentences over an algebraic number field is in

* *Keywords: Arithmetical sentences, computational complexity, fields. 1991 AMS Mathematics Subject Classification.* Primary 68Q40, 68Q25, 13G05

† Supported by a grant from the National Science Council of Republic of China

P;

2. The decision problem of $\forall\exists$ sentences over fields with characteristic 0 is in P;

3. The decision problem of $\forall\exists$ sentences over a function field with characteristic p is polynomial time reducible to the factorization of polynomials over Z_p.

4. The decision problem of $\forall\exists$ sentences over fields with characteristic p is polynomial time reducible to the factorization of polynomials over Z_p.

5. The decision problem of $\forall\exists$ sentences over all fields is polynomial time reducible to the factorization of integers over Z and the factorization of polynomials over finite fields.