A Fuzzy Logic of Final Number of Experts*

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The fuzzy sets theory is one of modern fields of artificial intelligence. The notion of fuzzy set was originally introduced in [1]. This concept is a base of fuzzy logic and its applications in fuzzy control and approximate reasoning.

Now t-norm and t-conorm are often used as fuzzy operations of conjunction and disjunction. Concepts of t-norm and t-conorm came to fuzzy logic from probability theory on metric spaces [2]. However when one uses these constructions as fuzzy conjunction and disjunction the truth values of resulting statements completely depend on the syntax of initial ones and do not take into account the semantic features of initial statements.

In the paper we present a fuzzy logic with operations which result depends on both syntactic and semantic features of the formulas.

Let us consider an algebraic system $< A, \sigma >$. Let $B = \{b_1, b_2 ..., b_k\}$ be a finite set of experts. Each expert has his own opinion about truth values of atomic sentences of the given signature σ. So each expert b_i may be represented by some elementary diagram D_{b_i} in the signature σ.

Summarizing information obtained from experts $b_1, b_2 ..., b_k$ we may introduce a mapping $Tr_a : S_a(\sigma) \to \wp(B)$ from the set $S_a(\sigma)$ of the atomic sentences of the signature σ into the set of all subsets of the set B of experts. For any $\varphi \in S_a(\sigma)$ we put $Tr_a(\varphi) = \{b \in B|$ the expert b suppose that φ is true $\}$.

According to the map Tr_a we define a mapping $Tr : S(\sigma) \to \wp(B)$ from the set $S(\sigma)$ of the sentences of the signature σ into the set of all subsets of the set of experts B as follows:

A1. If φ is atomic sentence then $Tr(\varphi) = Tr_a(\varphi)$

A2. If $Tr(\varphi) = B'$ and $B' \subseteq B$, then $Tr(\neg\varphi) = B \setminus B' = \overline{B'}$.

A3. If for sentences $\varphi, \psi \in S(\sigma)$ we have $Tr(\varphi) = B_1$ and $Tr(\psi) = B_2$, then $Tr(\varphi \& \psi) = B_1 \cap B_2, Tr(\varphi \vee \psi) = B_1 \cup B_2$ and $Tr(\varphi \Rightarrow \psi) = \overline{B_1} \cup B_2$.

A4. For sentences $\forall x \varphi(x)$ and $\exists x \varphi(x)$ we put $Tr(\forall x \varphi(x)) = \bigwedge\limits_{a \in A} \varphi(a)$ and $Tr(\exists x \varphi(x)) = \bigvee\limits_{a \in A} \varphi(a)$.

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Thus for each statement $\varphi \in S(\sigma)$ we determine an imaginable set $Tr(\varphi)$ of the experts who could vote for this statement. In particular, if all experts have voted for the given statement, then this statement corresponds to the set B. And if all experts have voted against given statement, then it corresponds to the empty set.

Now we present a fuzzy logic $\Im$ with truth values belong to the interval [0,1]. Let $\varphi \in S(\sigma)$ and $Tr(\varphi) = B'$. Then truth value $\mu(\varphi)$ of φ is computed by the following way:

$$\mu(\varphi) = \frac{||B'||}{||B||}.$$

Note that the negation in our fuzzy logic has the following property. For any sentence φ we have:

$$\mu(\varphi) + \mu(\neg\varphi) = 1.$$

Let us point out that the result of conjunction, disjunction and implication in our logic depends on semantics of sentences as well as on syntax.

Remark. Let sentences φ and ψ have truth values α_1 and α_2 respectively. Then the truth value of the sentence $(\varphi\&\psi)$ belongs to the interval $[max\{\alpha_1, \alpha_2\}, 1]$, the truth value of the sentence $(\varphi \vee \psi)$ belongs to the interval $[0, min\{\alpha_1, \alpha_2\}]$, and the truth value of the sentence $(\varphi \Rightarrow \psi)$ belongs to the interval $[max\{1 - \alpha_1, \alpha_2\}, 1]$.

Definition 1. The triple $A_B =< A, B, \sigma >$ is called *a model* of the logic $\Im$ if A is the universe of the systemA_B, σ is the signature of this system and B is a finite set of experts.

Note that in the case $||B|| = 1$ our definition concides with usual definition of algebraic system in classical logic.

Definition 2. The formula φ is called *the tautology of the logic* $\Im$ if for any model of the logic $\Im$ and for any interpretation of free variables of φ the truth value $\mu(\varphi) = 1$.

Theorem. A formula φ is a tautology of the logic $\Im$ iff it is a tautology of the classical logic.

References

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