Some results in the epsilon substitution method

Toshiyasu Arai
Graduate School of Science and Technology
Kobe University
Rokko-dai, Nada-ku, Kobe 657-8501
Japan
e-mail:arai@kurt.scitec.kobe-u.ac.jp

Abstract

Hilbert proposed the epsilon substitution method as a basis for consistency proofs. In the method we substitute finitary objects for transfinite expressions occurring in formal proofs. Typically, an epsilon term $\epsilon x.F[x]$ denotes some x satisfying $F[x]$ if such an x exists. Otherwise it denotes an arbitrary object. This is codifyed in Hilbert's transfinite axiom $F[t] \rightarrow F[\epsilon x.F]$. In order to show the consistency of the first order arithmetic, it suffices to find, given any finite set of transfinite axioms, a substitution (*solution*) which assigns numerical values to epsilon terms and under which all the axioms occurring in the given finite set are true.

Hilbert's Ansatz for finding a solving substitution for any given finite set of transfinite axioms is, starting with the null substitution S^0, to correct false values step by step and thereby generate the process $S^0, S^1, \ldots$. The problem is to show that the approximating process terminates. After Gentzen's innovation, Ackermann(1940) succeeded in proving the termination of the process for the first order arithmetic.

In this talk we expound basic ideas, report recent progress on the subject, and discuss its possible extensions.