The Logic of Paraconsistent Answer Sets

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Answer set programming (ASP) - based on the semantics proposed in [4] - has recently emerged as an interesting new approach to declarative programming for knowledge representation and AI problem solving, see eg [2]. In Pearce [7] it was shown how the nonclassical logic of here-and-there with strong negation, denoted in [9] and elsewhere by $\mathbf{N}_5$, can serve as a foundation for answer set programming. The main property involved is that answer sets can be viewed as a certain kind of minimal $\mathbf{N}_5$-model, called an *equilibrium* model. A second key property was established in [5]: programs are strongly equivalent wrt answer set semantics if and only if they are equivalent viewed as propositional theories in $\mathbf{N}_5$. Here, strong equivalence (wrt a given semantics) is defined as follows: two programs Π_1 and Π_2 are called *strongly equivalent* if for any program Π, $\Pi_1 \cup \Pi$ and $\Pi_2 \cup \Pi$ have the same semantics. This property shows that $\mathbf{N}_5$ can be used to reason about answer set programs, and $\mathbf{N}_5$-deduction may be relevant for program transformation and optimisation (see [9, 8]).

In this report we show how the paraconsistent version of answer set semantics also admits a natural underlying, monotonic logic, which we denote by $\mathbf{N}_9$, and we look at an alternative model theory for answer sets due to R. Routley. Paraconsistent answer sets (PAS) were studied as a logic programming semantics by Sakama and Inoue in [11]. Recently, Alcantara, Demasio and Pereira [1] have made some progress towards obtaining a logical, declarative style of characterisation for the PAS semantics. However, [1] do not axiomatise or otherwise syntactically characterise the underlying (monotonic) logic of PAS; nor do they investigate the problem of strong equivalence.

We consider the version of answer set semantics defined for disjunctive logic programs with two kinds of negation [4]. Strong (explicit) negation is denoted by '$\sim$' and the second, default negation, usually written as '*not*' will be denoted by '$\neg$'. The formulas of disjunctive programs therefore have the form

$$L_1 \wedge \ldots \wedge L_m \wedge \neg\, L_{m+1} \wedge \ldots \wedge \neg\, L_n \rightarrow K_1, \vee \ldots \vee K_k \qquad (1)$$

where each L_i, K_j is a literal (atom or strongly negated atom) and we may have $m = n$ and m or n may be zero. A logic program Π is a set of such formulas. The set of all ground literals in the language of Π is denoted by *Lit*. Since answer sets are defined for programs without variables, each formula of form (1) is treated as shorthand for the set of its ground instances. Let Π be a program without '$\neg$'. A *paraconsistent answer set (PAS)* of Π is a minimal (under set-theoretic inclusion) subset S of *Lit* such that

for each formula $L_1 \wedge \ldots \wedge L_m \rightarrow K_1 \vee \ldots \vee K_k$ of Π, if $L_1, \ldots, L_m \in S$ then, for some $i = 1, \ldots, k$, $K_i \in S$;

Now, let Π be an arbitrary program. For any set of ground literals $S \subset Lit$, the program Π^S is the program without '$\neg$' obtaining from Π by deleting (i) each formula containing a subformula $\neg L$ with $L \in S$, and (ii) all subformulas of the form $\neg L$ in the remaining formulas. A set S of ground literals is said to be a *PAS* of a program Π if and only if S is a PAS of Π^S.

We introduce a second negation $\neg$ into Routley semantics for paraconsistent Nelson logic $\mathbf{N}^-$ [10] and show that the logic $\mathbf{N}^-_\neg$ obtained in this way is identical with $\mathbf{N4}^\perp$ studied in [6]. Moreover, $\mathbf{N}^-_\neg$ is a conservative extension of $\mathbf{N}^-$ as well as of intuitionistic logic. The logic $\mathbf{N}_9$ is the $\mathbf{N}^-_\neg$-extension defined by Routley here-and-there models (RHT-models) containing only two starred and two unstarred worlds. Showing the equivalence of the frames of [1] and RHT-models we prove that PAS are characterized by a special kind of minimal RHT-model or equilibrium model. We prove also that logic programs are strongly equivalent iff they are equivalent as $\mathbf{N}_9$-theories.

Using the results of [6] we obtain algebraic semantics for $\mathbf{N}_9$, in particular, we show that this logic is characterized by the 9-element matrix, which explains the notation $\mathbf{N}_9$. It is proved that $\mathbf{N}_9$ is the minimal conservative extension of here-and-there logic in the lattice of $\mathbf{N}^-_\neg$-extensions, that it can be axiomatised over $\mathbf{N}^-_\neg$ by the formula $p \vee (p \to q) \vee \neg q$. Finally, we establish that $\mathbf{N}_9$ possesses the Craig interpolation property.

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