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On constructive nilpotent groups.*

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In this talk we obtain a criteria of constructibility of a two-stage nilpotent group and show that a constructivizable two-stage nilpotent torsion-free group is order constructivizable.

Let $\overline{G} = \langle G, \cdot, \leq, e \rangle$ be an ordered group and $\nu : \omega \to G$ be a numbering of the group $\overline{G}$. The system $\langle \overline{G}, \nu \rangle$ is called a *strongly constructive (constructive) ordered group*, if there exists an algorithm such that for any formula (atomic formula) $\Phi(x_0, \ldots, x_{n-1})$ of the language $L = \langle \cdot, \leq, e \rangle$ and for any numbers $m_0, \ldots, m_{n-1}$, it determines whether the property $\overline{G} \models \Phi(\nu m_0, \ldots, \nu m_{n-1})$ holds.

An ordered group $\overline{G}$ is called *(strongly) constructivizable*, if there exists a numbering ν such that the system $\langle \overline{G}, \nu \rangle$ is a (strongly) constructive ordered group. A group G is called *(strongly) order constructivizable* if there exist an ordering $\leq$ and a numbering ν such that the system $\langle \overline{G}, \nu \rangle$ is a (strongly) constructive ordered group. In [1], it was proved that every countable abelian torsion-free group, every free nilpotent group, every finitely generated nilpotent group, every finitely generated nilpotent torsion-free group and the group of unitary triangular matrixes over an associative ordered constructive ring with unit element are order constructivizable.

THEOREM 1 *Let (G, ν) be a constructive two-stage nilpotent group and B a computably enumerable subgroup of the center $Z(G)$ of the group G such that the quotient group G/B is an abelian torsion-free group. Then there exists a constructive numbering μ of the group G satisfying the following conditions:*

1) *there exists a computably enumerable basis of the subgroup B;*

2) *there exists a computably enumerable system of elemets $\{c_i \mid i \in I\}$ in (G, μ) such that the cosets $\{c_i + B\}$ form a basis of the quotient group A/B.*

COROLLARY 1 *Let (G, ν) be a constructive two-stage nilpotent group and $I(G')$ the isolator of the group commutant. Suppose that $I(G')$ is contained in the center of G. Then there exists a constructive numbering μ of the group G such that the subgroup $I(G')$ is computable in (G, μ).*

>From this result and the results of [1] we obtain

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COROLLARY 2 *A constructivizable two-stage nilpotent torsion-free group is order constructivizable.*

Assume that abelian groups A, B and a function $f : A \times A \longrightarrow B$ satisfy the following conditions: $A \cap B = \{e\}$, $f(a_0, e) = f(e, a_0) = f(a_0, a_o^{-1}) = f(a_0^{-1}, a_0) = e$, $f(a_0 a_1, a_2) f(a_0, a_1) = f(a_0, a_1 a_2) f(a_1, a_2)$, $a_i \in A$. We call the function f a *system of factors*. Define the group G as follows: $G = gr(A, B \mid\mid a_0 b_0 = b_0 a_0,\ a_0 b_0 \circ a_1 b_1 = a_0 a_1 f(a_0 a_1) b_0 b_1,$ $a_i \in A, b_i \in B)$. The group G is called an extension of the group B by A wrt the system of factors f. It is obvious that if (A, ν), (B, μ) are constructive abelian groups and a system of factors f is computable, then the natural numbering γ of the group G defined by ν and μ will be constructive.

COROLLARY 3 *Let G be a two-stage nilpotent group and let the isolator of the group commutant be contained in the center of G. Then G is constructivizable if and only if it is isomorphic to an extension of a constructive abelian group by a constructive abelian torsion-free group wrt a computable system of factors.*

COROLLARY 4 *A two-stage nilpotent torsion-free group is constructivizable if and only if it is isomorphic to an extension of a constructive abelian group by a constructive abelian group wrt a computable system of factors.*

>From the results proved in [2] we obtain one condition which follows from constructibility of a nilpotent torsion-free group.

PROPOSITION *Let G be a constructive nilpotent torsion-free group. Then there exists a central series of constructivizable subgroups*

$$R = G_0 \subseteq G_1 \subseteq \ldots \subseteq G_n = G,$$

such that all quotients G_{n+1}/G_n are constructivizable.

References

[1] *Romankov V.A., Khisamiev N.G. On constructive matrix groups and ordered groups, Algebra i logika, 2004, 3(43), 353—363 (in Russian).*

[2] *Khisamiev N.G. Hierarchies of torsion-free abelian groups, Algebra i Logika, 1986, 2(25), 128—142 (in Russian).*