First-order definability in the elementary type semigroup of superatomic Boolean algebras with distinguished subalgebra. *

Dmitry E. Pal'chunov
palch@math.nsc.ru
Institute of Mathematics, Novosibirsk, Russia

We consider Boolean algebras with distinguished subalgebra (S-algebras) in the signature $< \cup, \cap, \neg, 0, 1, S >$, where S is an unary predicate distinguishing a subalgebra.

An S-algebra $\boldsymbol{B}$ is called as ω-mixing of S-algebras $\boldsymbol{A}_1, ..., \boldsymbol{A}_n$ if $\boldsymbol{B}$ is isomorphic to the direct sum of S-algebras $\boldsymbol{C}_m,\ m \in N$, where for every $m,\ \boldsymbol{C}_m \cong \boldsymbol{A}_1 \times ... \times \boldsymbol{A}_n$.

Let $\mathbf{F}$ be the class of finite Boolean algebras with two-element subalgebras. Denote by $\mathbf{D}$ a minimal class of S-algebras containing the class $\mathbf{F}$ and closed under ω-mixing.

REMARK 1. Every S-algebra belonging to $\mathbf{D}$ is superatomic.

The class $\mathbf{D}$ of S-algebras was studied in [1].

Let $E \rightleftharpoons \{\boldsymbol{A} | \boldsymbol{A} \in \mathbf{D}\}/_{\equiv}$ be a set of elementary types of S-algebras from $\mathbf{D}$. Denote $\mathbf{E} \rightleftharpoons (E, \times)$, where $\times$ is a direct product: $[\boldsymbol{A}]_{\equiv} \times [\boldsymbol{B}]_{\equiv} = [\boldsymbol{A} \times \boldsymbol{B}]_{\equiv}$.

REMARK 2. $\mathbf{E}$ is a commutative semigroup having a unit. The unit of $\mathbf{E}$ is an elementary class consisting of degenerate S-algebra in which $0 = 1$.

The semigroup of elementary types of Boolean algebras with distinguished ideals was investigated in [2], [3].

THEOREM. A property of being finitely axiomatizable S-algebra is first-order definable in the semigroup $\mathbf{E}$. It means that there exists a first-order formula $\varphi(x)$ of the language $\{\times, =\}$ such that for any S-algebra $\boldsymbol{A} \in \mathbf{D}$, $Th\boldsymbol{A}$ is finitely axiomatizable iff $\mathbf{E} \models \varphi([\boldsymbol{A}]_{\equiv})$.

QUESTION. Does the statement of Theorem hold true for semigroup of elementary types of all S-algebras?

References

[1] D.E.Pal'chunov. Subalgebras of superatomic Boolean algebras. II International Conference on Algebra, Barnaul, 1991. Novosibirsk, 1991, p. 102.

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[2] D.E.Pal'chunov. Direct summands of Boolean algebras with distinguished ideals. Algebra and Logic, Plenum Pabl.Corp., v.31, N 5, 1992, 499-537.

[3] D.E.Pal'chunov. Elementary types semigroup of Boolean algebras with distinguished ideals. III International Conference on Algebra, Krasnoyarsk, 1993, p. 253.