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Introduction to New Parallel Computer Arithmetics Grounded on Factorizations of Operands

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Abstract. New arithmetics (*F*-arithmetics) doing the parallel process of multiplying and division is offered. The development of this arithmetics can give a scoring in accuracy of representation of numbers. However because of difficulty of execution of the operation of addition and code conversion *F*-arithmetics is classed as specialized arithmetics.

1 Introduction

Because of necessity of execution of intensive arithmetic calculations for the tasks of a digital signal processing, criptography, neural-like networks, interest to logarihtmic and residue number systems considerably has increased [1]—[4]. Recently computer arithmetics has received further development as double-dase number systems [5].

In [6] [7] parallel computer arithmetics grounded on the theorem of a number theory about uniqueness of decomposition of numbers on simple factors was offered. The factorization of operands allows completely to make by the parallel operation of multiplying and division of integers.

2 Representation of the positive integer numbers

We use the known items of information from a number theory (Sieve of Eratosthenes) that in an interval

$$\sqrt{N} < p_i \le N \ . \tag{1}$$

There can be only one prime number p_i, included in structure of canonical decomposition of an integer N:

$$N = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n} \ , \tag{2}$$

where $p_1, p_2, p_3 \ldots$ – simple factors $2, 3, 5\ldots$.

The generalization (1) gives

$$\sqrt[t+1]{N} < p_{\chi_1}^{(t)}, p_{\chi_2}^{(t)}, \ldots, p_{\chi_t}^{(t)} \le \sqrt[t]{N}\ , \qquad (3)$$

where $p_{\chi_1}^{(t)}, p_{\chi_2}^{(t)}, \ldots, p_{\chi_t}^{(t)}$ – simple factors from (2),

t — number of an interval $\left(\lfloor \log_2 N \rfloor \ge t \ge 1\right)$.

With registration (3) structure of number N is possible to present as:

$$\underbrace{\overbrace{2, \ldots, 2}^{1}}_{k=\lfloor \log_2 N \rfloor}, \ldots, \underbrace{\overbrace{p_{\chi_2}^{(i)}, \ldots, p_{\chi_i}^{(i)}}^{\pi]\sqrt[i+1]{N},\, \sqrt[i]{N}]}}_{i}, \ldots, \underbrace{\overbrace{p_{\chi_1}^{(2)}, p_{\chi_2}^{(2)}}^{\pi]\sqrt[3]{N},\, \sqrt{N}]}}_{2}, \underbrace{\overbrace{p_{\chi_1}^{(1)}}^{\pi]\sqrt{N},\, N]}}_{1}\ ,$$

where in $\underbrace{\overbrace{X \ldots X}^{\pi]a,\, b]}}_{t}$ $\pi]a, b]$ — function of Chebishev (amount of prime numbers existing in an interval $]a, b]$); t — maximum quantity of simple factors which are included in decomposition N in an interval $]a, b]$.

If not to take into account numbers of intervals, for which $\pi]a, b] = 0$, the decomposition N can be presented:

$$N = \underbrace{2^{\alpha_1} p_2^{\alpha_2} \cdots p_j^{\alpha_j}}_{r+1}\ \underbrace{p_{\chi_2}^{(k-i)} \cdots p_{\chi_{k-i}}^{(k-i)}}_{r=k-i} \cdots \underbrace{p_{\chi_1}^{(2)} p_{\chi_2}^{(2)}}_{2}\ \underbrace{p_{\chi_1}^{(1)}}_{1}\ , \qquad (4)$$

where r — rank of *F*-arithmetics, $j \le i$. As sequence of prime numbers $2, p_2, \ldots, p_j$ is known, representation of number N is possible to write as:

$$N = \left(\underbrace{p_{\chi_1}^{(1)}}_{1}; \underbrace{p_{\chi_2}^{(2)}, p_{\chi_1}^{(2)}}_{2}; \ldots; \underbrace{p_{\chi_{k-i}}^{(k-i)}, \ldots, p_{\chi_2}^{(k-i)}}_{r=k-i}; \ldots; \underbrace{\alpha_j, \ldots, \alpha_2, \alpha_1}_{r+1} \right)_F . \qquad (5)$$

The main parameters (5) are selected for $1 \le N \le N_{\max}$.

Example 1. Let $N_{\max} = 2^8 - 1$. The application (3) will give the following outcomes:

$$1)\ \sqrt{255} < \{17, 19, \ldots, 251\} \le 255;$$

$$2)\ \sqrt[3]{255} < \{7, 11, 13\} \le \sqrt{255};$$

$$3)\ \sqrt[4]{255} < \{5\} \le \sqrt[3]{255};$$

$$4)\ \sqrt[5]{255} < \{\varnothing\} \le \sqrt[4]{255};$$

$$5)\ \sqrt[6]{255} < \{3\} \le \sqrt[5]{255};$$

$$6)\ \sqrt[7]{255} < \{\varnothing\} \le \sqrt[6]{255};$$

$$7)\ \sqrt[8]{255} < \{2\} \le \sqrt[7]{255}.$$

The example of representation of numbers 231 … 255 with the help (5) at $r = 1$ is figured in the table 1. The absent factors $p_{\chi_1}^{(1)}$ (that is $p_{\chi_1}^{(1)} = 1$) are conditionally designated in zero. As the a matter of convenience zero terms are not eliminated.

Table 1. Example of representation of numbers 231 … 255 in *F*-arithmetics for $N_{\max} = 2^8 - 1$.

N	$p_{\chi_1}^{(1)}$	α_6	α_5	α_4	α_3	α_2	α_1
231			1	1		1	
232	29						3
233	233						
234		1				2	1
235	47				1		
236	59						2
237	79					1	
238	17			1			1
239	239						
240					1	1	4
241	241						
242			2				1
243						5	
244	61						2
245				2	1		
246	41					1	1
247	19	1					
248	31						3
249	83					1	
250					3		1
251	251						
252				1		2	2
253	23		1				
254	127						1
255	17				1	1	

Example 2. Let $N_{\max} = 2^{16} - 1$. The application (3) will give the following outcomes:

1) $\{257, 263, \ldots, \}$; 9) $\{\varnothing\}$;

2) $\{41, 43, \ldots, 251\}$; 10) $\{3\}$;

3) $\{17, 19, \ldots, 37\}$; 11) $\{\varnothing\}$;

4) $\{11, 13\}$; 12) $\{\varnothing\}$;

5) $\{7\}$; 13) $\{\varnothing\}$;

6) $\{5\}$; 14) $\{\varnothing\}$;

7) $\{\varnothing\}$; 15) $\{2\}$.

8) $\{\varnothing\}$;

3 Execution of arithmetic operations

As the representation (5) is created on the basis of a factorization of numbers, the operations of integer multiplying and division are most conveniently fulfilled. Rules of execution of multiplying:

1) $\left(\alpha'_j, \ldots, \alpha'_2, \alpha'_1\right) \times \left(\alpha''_j, \ldots, \alpha''_2, \alpha''_1\right) = \left(\alpha'_j + \alpha''_j, \ldots, \alpha'_2 + \alpha''_2, \alpha'_1 + \alpha''_1\right)$,

2) $\left(p'^{(t)}_{\chi_{k-i}}, \ldots, p'^{(t)}_{\chi_2}\right) \times \left(p''^{(t)}_{\chi_{k-i}}, \ldots, p''^{(t)}_{\chi_2}\right) =$
$= \text{Drop}[\text{Sort}[p'^{(t)}_{\chi_{k-i}}, \ldots, p'^{(t)}_{\chi_2}, p''^{(t)}_{\chi_{k-i}}, \ldots, p''^{(t)}_{\chi_2}], t]$.

In the item 2 the functions of the "mathematician" environment are used [8]. For $t = 1$

$$\left(p'^{(1)}_{\chi_1}\right)_F \times \left(p''^{(1)}_{\chi_1}\right)_F = \left(p'^{(1)}_{\chi_1}\right)_2 \oplus \left(p''^{(1)}_{\chi_1}\right)_2,$$

where the character $\oplus$ the operation of addition module 2.

4 Conclusion

Thus obtained arithmetics can be classed as parallel arithmetics. The independence of processing of small bit digits of F-representation specifies convenience of usage of tabulared devices. Disadvantages of F-arithmetics is the difficulty of execution of the operation of addition and code conversion. However for large $N_{\max}$ it is possible to use a polynomial number system with the large basis $q = N_{\max} + 1$:

$$N = \sum_{i=1}^{n} a_i \left(N_i\right)_F q^i ,$$

where $a_i \in \{0, 1\}$. The generalization of F-arithmetics on area problem-oriented arithmetics and codes is given in [9].

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