DEVELOPMENT OF TURBULENT CIRCULATION FROM AN AREA HEAT SOURCE IN STABLE STRATIFIED ENVIRONMENT*

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A three-equation model of the turbulent transport of momentum and heat for simulating a circulation structure over the heat island in a stably stratified environment under nearly calm conditions is formulated. The turbulent kinetic energy (TKE), $E=(1/2)<u_iu_i>$, its spectral flux ε (dissipation), and the dispersion of turbulent fluctuations of temperature $<\theta^2>$ are found from differential equations, thus the correct modeling of transport processes in the interface layer with the counter-gradient heat flux is assured. Turbulent fluxes of momentum $-<u_iu_j>$ and heat $-<u_i\theta>$ are determined from E–ε–$<\theta^2>$ turbulence model minimizes difficulties in simulating the turbulent transport in a stably stratified environment and reduces efforts needed for the numerical implementation of the model. Numerical simulation of the turbulent structure of the penetrative convection over the heat island under conditions of stably stratified atmosphere demonstrates that the three-equation model is able to predict the circulation induced by the heat island, temperature distribution, root-mean-square fluctuations of the turbulent velocity and temperature fields, and spectral turbulent kinetic energy flux that are in good agreement with the experimental data and results of LES.

1 Introduction

Turbulence closure models are often used as tools to analyze and predict atmospheric boundary layer characteristics. During the last 20 years numerous articles dealing with various types of flows using different models have been presented. For stratified atmospheric flows the most frequently used models are E–ε models (Duynkerke 1988), second-order closure models (Zeman and Lumley 1979; Sun and Ogura 1980) and third-order closure models (Andre et al. 1978; Canuto et al. 1994). Together with large eddy models (Moeng 1984; Mason 1989; F.T.M. Nieuwstadt et al. 1993) third-order closure models (Andre et al. 1978) should be considered as fundamental research tools because of their large computer demands. A growing need for detailed simulations of turbulent structures of stably stratified flows motivates the development and verification of computationally less expensive closure models for applied research that should be kept as simple as possible in order to reduce computational demands to a minimum. The ideas underlying the algebraic models represent an improvement in buoyant flow modeling and could be used for applied modeling since a full second-order closure model is presently much too demanding. Indeed, the recent studies (Andren 1991; Sommer and So 1995) of the stable stratified flows indicate that a model with a transport approximation including buoyancy effects might be the optimal way that combines both computational efficiency and predictive capability. The algebraic modeling techniques of previous studies could be modified to obtain an algebraic heat-flux model for buoyant flows. In order to avoid using the symbolic algebra software for inverting a system of algebraic equations for the turbulent heat fluxes $-<u_i\theta>$ and turbulent momentum fluxes $-<u_iu_j>$ it is desirable to derive an explicit algebraic heat-flux model here the heat fluxes are expressed explicitly in terms of the mean gradients and the eddy diffusivities. It should be pointed out that the use in higher-order closure studies of the ε-equation is now quite standard (Canuto et al. 1994; Ilyushin and Kurbatskii 1997). Results of computational modeling and simulation of the atmospheric boundary layer (Andren 1991; Ilyshin and Kurbatskii 1996) showed the importance of retaining the full prognostic equation for the temperature variance, allowing a counter-gradient transport of heat in the upper half of the turbulent layer.

The present paper proposes and evaluates a turbulence closure scheme that has been implemented in order to make the model more useful for stable stratified flows and air pollution applications.

In this study the E–ε–$<\theta^2>$ model is applied. In the model the eddy-exchange coefficients are evaluated from the turbulent kinetic energy E and the viscous dissipation ε. The turbulent fluxes $-<u_i\theta>$ and $-<u_iu_j>$ are cal-

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culated from fully explicit algebraic models for the penetrative turbulent convection from an area heat source (the urban heat island) with no initial momentum under calm and stably stratified environment. The performance of the three-equation model was tested by comparing the computed results with the laboratory measurements of the low-aspect-ratio plume (Lu et al. 1997) and the LES data (Canuto et al. 1994). Good agreements were found.

2 The heat-island mathematical model

A simple theoretical model of the nocturnal urban heat island cannot be applied to the case of near calm conditions when the ambient wind speed approaches zero, because in low-aspect-ratio plumes ($z_i / D << 1$, where z_i is the mixing height and D is the heat island diameter) the basic assumptions of classical plume theory are violated (Lu et al. 1997).

In most mathematical models the thermal plume turbulence is parameterized (cf. Byun and Araya 1990). However, to analyze and understand the turbulent structure of the urban-heat-island phenomenon and its associated circulation, the turbulence modeling is required.

The penetrative turbulent convection is induced by the constant heat flux H_0 from the surface of a plate with diameter D (Figure 1). It simulates a prototype of an urban heat island with the low-aspect-ratio plume ($z_i / D << 1$) under near calm conditions and stably stratified atmosphere. The flow is assumed to be axisymmetric. In the experiment the mixing height, z_i, is defined as a height where the maximum negative difference between the temperature in the center of the plume and the ambient temperature T_a is achieved as shown in Figure 1, and z_i includes the interface layer in the upper part of the plume.

Analysis of the experimental data (Lu et al. 1997) leads to the following choice of the characteristic scale for the horizontal (radial) velocity:
$W_D = (\beta g D H_0 / \rho c_p)^{1/3}$, $\beta = -(1/\bar{\rho})\ (\partial\rho/\partial T)_p$ is the thermal expansion coefficient, $\bar{\rho}$ – mean density, c_p – specific heat at constant pressure. For the heat island with a low-aspect-ratio of the vertical linear scale to the horizontal linear scale D the continuity equation yields that the characteristic vertical linear scale is of the order $(z \cdot Fr)$, and the characteristic scale for the vertical velocity has the value $(W_D Fr)$ where $Fr = W_D /(N \cdot D)$ is the Froude number, $N = \left[\beta g (\partial T / \partial z)_a\right]^{1/2}$ is the Brunt-Vaisala frequency. $(W_D \cdot N)/(\beta g)$ is taken to be the characteristic temperature scale, and D / W_D is the time scale.

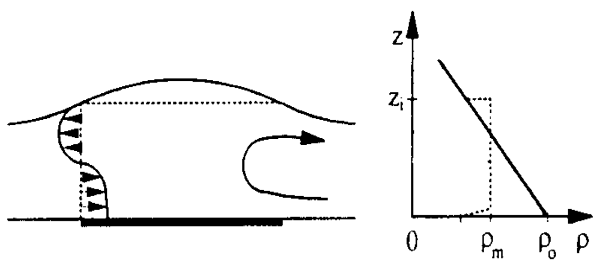

Figure 1a. Schematic diagram of the heat-island circulation including horizontal velocity distribution and vertical density profiles (z_i – mixing height, ρ_0 – density of the reference atmosphere, ρ_m – plume centerline density).

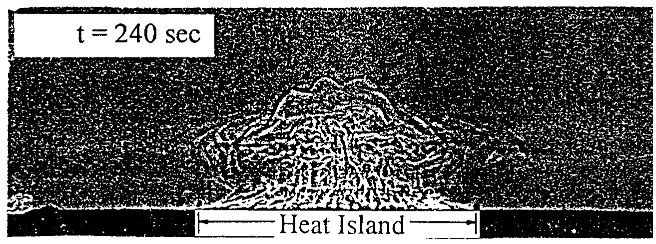

Figure 1b. Shadowgraph picture of the heat island. At t = 240 sec the full circulation is established.

2.1 **Governing equations.** Fundamental fluid dynamics equations describing the circulation over the low-aspect-ration heat island can be written in the hydrostatic approximation (Pielke 1984). In the absence of the Coriolis force and radiation, the governing equations in non-dimensional form for mean values of velocity and temperature in Boussinesq approximation are,

$$\frac{\partial V}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rV)V+\frac{\partial}{\partial z}VW=\int_z^H \frac{\partial T}{\partial r}dz-\frac{\partial < v^2 >}{\partial r}-$$

$$-\frac{\partial <vw>}{\partial r}+\frac{<u^2>-<v^2>}{r}+\mathrm{Re}^{-1}\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial V}{\partial r}-\frac{V}{r^2}\right)+\mathrm{Fr}^{-2}\mathrm{Re}^{-1}\frac{\partial^2 V}{\partial z^2}, \quad (1)$$

$$\frac{\partial}{\partial r}(rV)+\frac{\partial}{\partial z}(rW)=0, \quad (2)$$

$$\frac{\partial T}{\partial r}+\frac{1}{r}\frac{\partial}{\partial r}(rVT)+\frac{\partial}{\partial z}(WT)=\mathrm{Re}^{-1}\mathrm{Pr}^{-1}\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial T}{\partial r}++\mathrm{Fr}^{-2}\frac{\partial^2 T}{\partial z^2}\right)+\left\{-\frac{1}{r}\frac{\partial}{\partial r}r<v\theta>-\frac{\partial}{\partial z}<w\theta>\right\}. \quad (3)$$

In equations (1) - (3) V is the mean horizontal velocity, W – mean vertical velocity, v – horizontal turbulent velocity fluctuation, w – vertical turbulent velocity fluctuation, u – azimuthal turbulent velocity fluctuation, T – mean temperature, θ – turbulent temperature fluctuation, $\mathrm{Re}=(W_D D)/\nu$ – Reynolds number, $\mathrm{Pr}=\nu/k$ – Prandtl number, k – thermal diffusivity coefficient, ν – kinematic viscosity, H – given height of the stratified layer. In (1) - (3) and everywhere in the following discussion capital letters and brackets $<\ldots>$ define mean values of variables, and lower-case letters are reserved for turbulent fluctuations.

2.2 **The explicit algebraic model of turbulent fluxes.** A physically correct description of the effect of stable stratification on the circulation over the heat island can be obtained by using a three-equation turbulence transport model. The TKE, its dissipation ε and the dispersion of turbulent fluctuations of temperature $<\theta^2>$ are found from the differential transport equations, and the turbulent fluxes of momentum $-<u_i u_j>$ and heat $-<u_i\theta>$ are determined from fully explicit algebraic "gradient diffusion" equations. This three-equation turbulence model minimizes difficulties in describing turbulence in stable stratified flow and reduces efforts required for its numerical implementation.

The explicit algebraic model for the turbulent heat flux vector $-<u_i\theta>$ can be derived from exact transport equations (Kurbatskii 1988, Sommer and So 1995) in the approximation of equilibrium turbulence

$$-<u_i\theta>=\frac{\sqrt{\tau\cdot\tau_\theta}}{C_{1\theta}}\{[<u_i u_j>\frac{\partial T}{\partial x_j}+(1-C_{2\theta})<u_j\theta>\frac{\partial U_i}{\partial x_j}]+(1-C_{2\theta})g_i\beta<\theta^2>\}, \quad (4)$$

where $\tau=E/\varepsilon$, $\tau_\theta=<\theta^2>/2\varepsilon_\theta$ are characteristic time scales, $\varepsilon_\theta=2k<\partial\theta/\partial x_j\cdot\partial\theta/\partial x_j>$ is the destruction of temperature fluctuations. $C_{1\theta}, C_{2\theta}$ are constant coefficients, their values are given below.

The expression (4) constitutes a linear system of algebraic equations for $<u_i\theta>$. This expression turns out to be implicit for flux $-<u_i\theta>$ because $<u_j\theta>$ is included into the right-hand side of (4). The easiest way to obtain a fully explicit model for $-<u_i\theta>$ is to use the gradient transport model for the fluxes $-<u_i u_j>$ and $-<u_j\theta>$ in the right-hand-side of (4):

$$-<u_i u_j>=2\nu_T S_{ij}-(2/3)E\delta_{ij}, \quad (5)$$

$$-<u_j\theta>=k_T(\partial T/\partial x_j), \quad (6)$$

where $S_{ij}=(1/2)(\partial U_i/\partial x_j+\partial U_j/\partial x_i)$ is the mean strain tensor, $\nu_T=C_\mu E^2/\varepsilon$ – turbulent viscosity, $k_T=C_T\sqrt{2R}E^2/\varepsilon$ – turbulent thermal diffusivity, $R=\tau_\theta/\tau$ – ratio of the characteristic scales of temperature(τ_θ) and dynamic (τ) turbulent fields. Coefficients in (4) – (6) have "standard" values calibrated by modeling the homogeneous turbulence in stable stratified flows (e.g. Sommer and So, 1995): $C_\mu=0.09$, $C_T=0.095$, $C_{1\theta}=2.32$, $C_{2\theta}=0.4$.

Substitution of (5) and (6) into (4) gives the fully explicit algebraic model for the turbulent heat flux vector:

$$-<u_i\theta>=C_T\frac{E^2}{\varepsilon}\sqrt{2R}\frac{\partial T}{\partial x_i}-\frac{\sqrt{R}}{C_{1\theta}}\frac{E}{\varepsilon}[\{2\nu_T+(1-C_{2\theta})\times$$

$$\times k_T\}S_{ij} + (1 - C_{2\theta})k_T\Omega_{ij}](\partial T/\partial x_j) + +[(1 - C_{2\theta})/C_{1\theta}] \times (E/\varepsilon)\cdot\sqrt{R}g_i\beta < \theta^2 >, \tag{7}$$

where $\Omega_{ij} = (1/2)(\partial U_i/\partial x_j - \partial U_j/\partial x_i)$ is the mean rotational tensor, $R = 0.6$.

For normal turbulent stresses in the right-side of (1) the present work adopts a simple Boussinesq model that preserves certain anisotropy of the normal stresses,

$$< v^2 >= (2/3)E - 2\nu_T(\partial V/\partial r), \tag{8}$$

$$< w^2 >= (2/3)E - 2\nu_T(\partial W/\partial z), \tag{9}$$

$$< u^2 >= (2/3)E - 2\nu_T(V/r). \tag{10}$$

For the shear stress the Boussinesq model (5) yields:

$$- < vw >= (2/3)E - 2\nu_T(\ \partial V/\partial z + \partial W/\partial r). \tag{11}$$

Substitution of (8) – (11) into (1) leads to the close form of the equation for the mean radial velocity V. The vertical mean velocity W is then found as a quadrature from the continuity equation (2). Results of the modeling discussed below in Section 4 show that the proposed fully explicit algebraic model for turbulent fluxes of momentum and heat (5) – (7) provides quite acceptable predictions of the structure of such a complex phenomenon as the heat island in stratified environment.

Quantities E, ε and $<\theta^2>$ in the three-equation model are found from the differential equations of turbulent transport. Equations for E and ε in non-dimensional form are,

$$\frac{\partial E}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rVE) + \frac{\partial}{\partial z}(WE) = \frac{1}{r}\frac{\partial}{\partial r}\{r[\mathrm{Re}^{-1} + \frac{\nu_T}{\sigma_E}]\frac{\partial E}{\partial r}\} + \frac{\partial}{\partial z}\{[\mathrm{Re}^{-1}Fr^{-2} + \frac{\nu_T}{\sigma_E}]\frac{\partial E}{\partial z}\} + P - \varepsilon + G, \tag{12}$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rV\varepsilon) + \frac{\partial}{\partial z}(W\varepsilon) - \frac{1}{r}\frac{\partial}{\partial r}\{r[\mathrm{Re}^{-1} + \frac{\nu_T}{\sigma_\varepsilon}]\frac{\partial \varepsilon}{\partial r}\} - -\frac{\partial}{\partial z}\{[\mathrm{Re}^{-1}Fr^{-2} + \frac{\nu_T}{\sigma_\varepsilon}]\frac{\partial \varepsilon}{\partial z}\} = -\frac{\varepsilon}{\tau}\cdot\Psi, \tag{13}$$

where $P = \nu_T[Fr^{-2}(\partial V/\partial z)^2 + Fr^{-1}(\partial V/\partial z)(\partial W/\partial r)] - (2/3)E(\partial V/\partial r + \partial W/\partial z)$ is the TKE production due to shear, $G =< w\theta >$ – TKE production due to the buoyancy force fluctuation, σ_E and σ_ε – turbulent Prandtl numbers ($\sigma_E = 1.0$, $\sigma_\varepsilon = 1.3$). Function Ψ is written in the form,

$$\Psi = \Psi_0 + \Psi_1\cdot b_{ij}\cdot q^2(\partial U_i/\partial x_j)/\varepsilon + \Psi_2\beta\ g_i < \theta u_i >$$

$$/\varepsilon + \Psi_3\beta g_i < \theta u_i > /\varepsilon q^2(\partial U_i/\partial x_j)/\varepsilon. \tag{14}$$

In (14) $b_{ij} \equiv< u_i u_j > /q^2 - \delta_{ij}/3$, $q^2 = 2E$ and coefficients $\Psi_0, \Psi_1, \Psi_2, \Psi_3$ have "standard" values (e.g. Andren 1991) calibrated by solving different atmospheric boundary layer problems: $\Psi_0 = 3.8$; $\Psi_1 = 2.4$; $\Psi_2 = -2.4$; $\Psi_3 = 0.3$.

Equation for the dispersion of turbulent temperature fluctuations $<\theta^2>$ is written in the following close form,

$$\frac{\partial <\theta^2>}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}rV <\theta^2> + \frac{\partial}{\partial z}W <\theta^2> = \frac{1}{r}\frac{\partial}{\partial r}[r(C_{\theta 2}\frac{E}{\varepsilon} < v^2 > \frac{\partial <\theta^2>}{\partial r})] + \frac{\partial}{\partial z}[C_{\theta 2}\frac{E}{\varepsilon}\times$$

$$\times < w^2 > \frac{\partial <\theta^2>}{\partial z}] - 2 < v\theta > \frac{\partial T}{\partial r} - 2 < w\theta > \frac{\partial T}{\partial z} - \frac{1}{R}\cdot\frac{\varepsilon}{E} < \theta^2 >. \ (C_{\theta 2} = 0.11) \tag{15}$$

2.3 **Boundary conditions.** The problem of the development and evolution of circulation above the heat island is assumed to be axisymmetric. The domain of integration is a cylinder of a given height H. The heated plate with diameter D is located at the center of the cylinder bottom (Figure 1). The outer boundary is located at the distance $R \cong 1.5D$ from the cylinder axis. At the initial moment the medium is at rest and it is stably stratified.

Conditions

$V = (\partial E / \partial r) = (\partial \varepsilon / \partial r) = (\partial T / \partial r) = \partial < \theta^2 > / \partial r = 0$ are prescribed at the plume axis ($r = 0$) and at its outer boundary ($r = R$).

At the top boundary ($z = H$) the zero-flux condition is enforced,

$$\partial V / \partial z = \partial E / \partial z = \partial \varepsilon / \partial z = \partial < \theta^2 > / \partial z = 0 .$$

Boundary condition for the temperature at the top boundary is written so that the vertical temperature gradient is the same at two last mesh points,

$$(\partial T / \partial z)_{z=z_{J-1}} = (\partial T / \partial z)_{z=z_J} .$$

For the horizontal mean velocity on the underlying surface the no-slip condition is specified, $V\big|_{z=0} = 0.$ The surface heat source on the bottom boundary ($z = 0$) has the size $0 \le r / D \le 0.5$. It prescribes non-homogeneous boundary conditions for E, ε, T and $< \theta^2 >$.

Values of E and ε for $0 \le r / D \le 0.5$ at the first mesh point ($z = z_1$) are taken to be (Panofsky et al. 1977), $E_1 = u_*^2[\{7 + 0.52(z_i / L)\}^{2/3} + 0.85\{1 + 3(z_i / L)\}^{2/3}]$, $\varepsilon_1 = (u_*^3 / L) \cdot \{1 + 0.5(z_i / L)^{2/3}\}^{3/2}$; u_* – friction velocity on the underlying surface evaluated from the experimental data (Lu et al. 1997), its value was $u_* / W_D \cong 0.045$. The value $L = u_*^3 /(\kappa \beta g \mathrm{H}_0 / \rho c_p)$ is the Obukhov – Monin scale ($\kappa = 0.40$ – *Karman* constant).

Outside the surface heat source ($0.5 \le r / D \le R / D$) values of E and ε at the first mesh point are chosen according to Andre et al. (1978) as,

$$E_1 = C_\mu^{-1/2} u_*^2 ,\ \varepsilon_1 = (u_*^3 / \kappa)(Fr^{-1} z_1^{-1} + 4 / L) .$$

For $0.5 \le r / D \le R / D$ temperature is taken to be equal to the surface temperature, $T\big|_{z=0} = T_w$. On the surface of the source $0 \le r / D \le 0.5$ the heat flux H_0 is prescribed.

Boundary condition for the dissipation $< \theta^2 >$ for $0 \le r / D \le 0.5$ at the first mesh point ($z = z_1$) is obtained from the equation (15) approximated as locally balanced ("production $\approx$ dissipation") and from the equation (7) for the turbulent fluxes:

$$< \theta^2 >_1 = = \frac{[(H_0 / \rho c_p) \beta g D / W_D^3]^2 (2R)^{1/2} Fr^2 /(C_T E_1)}{(1 + 2R(1 - C_{2\theta}) /(C_{1\theta} C_T \varepsilon_1)[H_0 / \rho c_p) \beta g D / W_D^3])} .$$

For $0.5 \le r / D \le R / D$ a background value of dispersion (at $z = z_1$) is specified as a function of the dispersion at the heated plate, $< \theta^2 (0, r / D > 0.5) >_1 \cong 10^{-2} < \theta^2 (0, 0 \le r / D \le 0.5) >_1$.

3 Discussion of results and concluding remarks

Figure 2 shows streamlines for positive values of the streamfunction ($r / D < 0$ – counterclock-wise circulation), and for negative values of the streamfunction ($r / D > 0$ – clockwise circulation).

Streamlines in Figure 2a (experiment) and Figure 2b (computation) are similar. They show the main upflow in the center generated by two vortices that reaches the interface layer ($z / z_i \propto 1$), and downflow in the outer region. Common feature for both experiment and computation is the suppression of the plume height by the stable stratification, the increase in the sideway flow and turbulence of the plume. The flow above the vortex pair (Figure 2b) can be explained by local circulation over the real-life heat island. In this situation the heat is not transported from the urban surface into the upper atmosphere, instead it is accumulated in local circulations.

a *b*

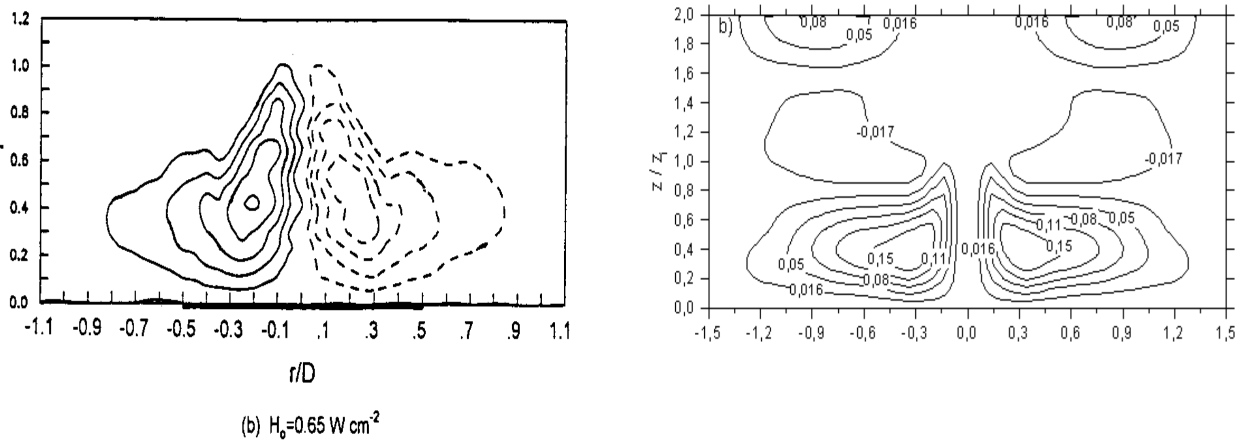

Figure 2. Streamline contours for $Fr = 0.077$, $Re = 8280$. a) – experimental data (Lu et al. 1997). Solid lines: counterclockwise motion. Dashed lines: clockwise motion. b) – simulation results.

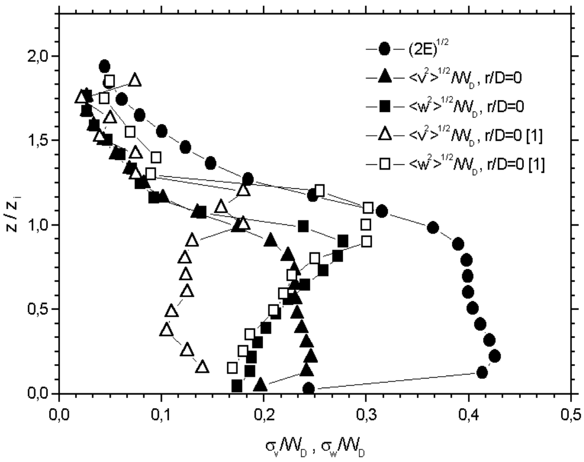

Figure 3. Nondimensional variances of velocity on z/z_i at center above the heat island. The laboratory data (Lu et al. 1997: $Fr = 0.077$; $Re = 8280$): △ – horizontal velocity profile, □– vertical velocity profile. The computation data: ▲ – horizontal velocity variance, ■ – vertical velocity variance; ● – computed intensity of turbulence, $q^2 = < u_i^2 >$.

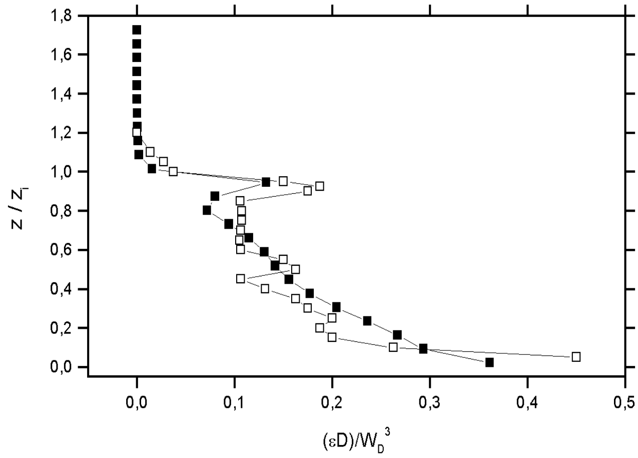

Figure 4. Turbulent kinetic energy dissipation ε normalized by $W_D^3 D^{-1}$ as a function of z/z_i . ■ – computation (at $r/D = 0.225$); □ – Moeng & Wyngaard's LES (redrawn from Canuto et al. 1994).

The turbulent plume structure is shown in Figure 3 as a distribution of root-mean-square fluctuations of vertical and horizontal turbulent velocities as functions of z/z_i at the plume center. It should be mentioned that the simple gradient model (8) and (9) not only correctly predicts characteristic features of σ_v/W_D and σ_w/W_D distributions, but also satisfactorily reflects their anisotropic nature.

Figure 4 shows comparisons of the vertical profile of the TKE dissipation, ε , computed by the three-equation model with LES data of Moeng and Wyngaard (redrawn from Canuto et al. 1994).

Figure 5 show that temperature profiles inside the plume have characteristic "swelling": the temperature inside the plume is lower than the temperature outside at the same height creating an area of negative buoyancy due to the over-shooting of the plume at the center. The height of the overshoot is maximum at the plume center and it decreases away

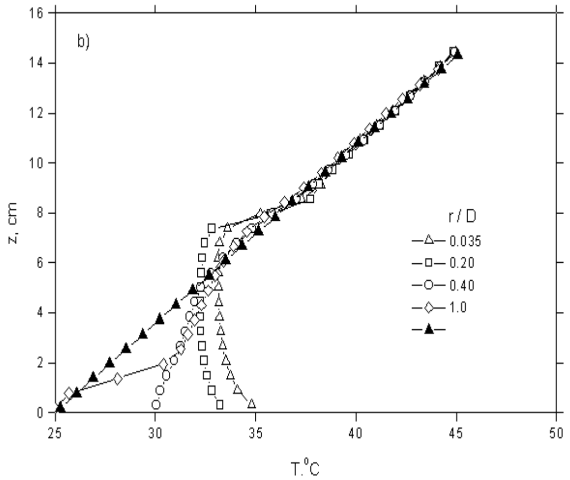

Figure 5. Computed mean temperature profile at various locations at $t = 8$ min ($\mathrm{Re} = 4500$, $\mathrm{Fr} = 0.088$) ; ▲ – the initial profile.

from the center. The temperature anomaly at the plume center extends to a higher height than at the cross-section $r / D = 0.20$. This behavior of the vertical temperature distribution indicates that the plume has a dome-shaped upper part in the form of a "hat" schematically shown in Figure 1a and also in figure 1b (the shadowgraph of Lu et al. 1997).

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