ON SOLUTION OF ONE INVERSE PROBLEM OF UNDERGROUND HYDROMECHANICS CONNECTED WITH OIL STRATUM PARAMETER ESTIMATION

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The numerical approach is proposed to mathematical processing of measurement data obtained during exploitation of an oil well. We consider the hydrodynamic method of oil stratum sounding that is based on measurement of the pressure and flow rate of a fluid in a borehole. An algorithm for solution of the inverse problem is based on application of minimization methods and the so-called objective functionals. The results of its evaluation on model data are presented.

Introduction

In this paper, we propose a numerical approach to mathematical processing of measurement data obtained during exploitation of an oil well. The proposed approach based on application of the theory of inverse problems of mathematical physics has showed itself to advantage in solution of a number of problems of exploration geophysics. Here we consider the hydrodynamic method of oil stratum sounding that is based on measurement of the pressure and flow rate of a fluid in a borehole [4, 5]. The input data are the *pressure-time curves* (PTC) which together with other information are obtained in large quantities during development and testing of oil wells.

The technology of investigation of a well is as follows:

1) a self-contained pressure gauge is installed at the wellhead in order to record the change of pressure versus time;

2) the pressure in the well is diminished with the aid of various devices in the course of the time interval t_v in the beginning of development of the unperturbed stratum;

3) after the pressure has been staying at low level (during the time interval t_v) the action at the stratum is stopped.

Then either the well is sealed (PRC technology; PRC stands for *pressure recovery curve*), or one stops to extract the fluid that flows in from the stratum and it accumulates in the borehole (LRC technology; *level recovery curve*). So, the level of the fluid rises and the wellhead pressure increases. The increase of pressure after the influence on the stratum has been terminated depends on the stratum parameters and processes in the borehole (interfering factors).

Useful information about the stratum parameters is obtained by analyzing the PRC or IC (*inflow curve* in the LRC method).

A mathematical model of the process taking into account all modern conceptions about the physics of its proceeding is very complicated and cannot be investigated and analyzed. Therefore, we consider a simplified model of the medium and process which, nevertheless, adequately represents main tendencies of the process' proceeding. Namely, we consider one-phase filtration in a horizontal stratum that is inhomogeneous in stretching and is bounded by a circular contour of feeding. The inhomogeneity of the stratum is assigned by the permeability function, i. e., all other parameters are assumed to be constant throughout the stratum. We assume

*This work was supported, in part, by the Russian Foundation for Basic Research, under grants No. 01–05–64704, 99–05–68372 and 00–15–99092.
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that the hydroconductivity and piesoconductivity of the stratum are linearly related to the permeability. One boundary condition assigns the known changing of the wellhead pressure and the other is used in computation of IC and PRC.

Under the above assumptions the model equations turn out to be relatively simple and numerical solution of the direct problem (to determine the pressure field in an inhomogeneous stratum during testing of a well under assumption that the stratum parameters are known) does not make difficulties when modern personal computers are used. There are algorithms for processing of PRC's that allow to determine the parameters of stratum areas lying far from a well [5], and these available algorithms are widely used in practice. However, of special interest for oil engineers and geophysicists is to determine the stratum parameters in the vicinity of a borehole. The source data for solution of this problem are the wellhead pressure change curve that is known from the field experiment, the pressure at the feeding contour, the hydroconductivity of the remote zone, and the curve showing the reduction of the wellhead pressure in time in the course of exerting action at the stratum (during swabbing, compressing, etc.).

In this paper, we propose an algorithm for solution of some of the above-mentioned inverse problems that is based on application of minimization methods and the so-called objective functionals. The results of its evaluation on model data are also presented.

1. Statement of the problem

Let p_1 be the pressure in the stratum for $r_c < r < R_1$ and p_2, for $R_1 < r < R_k$. Consider the system (cf. [8])

$$\frac{\partial p_1}{\partial t} = \chi_1 \cdot \left(\frac{\partial^2 p_1}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial p_1}{\partial r}\right), \quad r_c < r < R_1, \quad t > 0, \tag{1}$$

$$\frac{\partial p_2}{\partial t} = \chi_2 \cdot \left(\frac{\partial^2 p_2}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial p_2}{\partial r}\right), \quad R_1 < r < R_k, \quad t > 0, \tag{2}$$

with the initial and boundary conditions

$$p_1|_{t=0} = P_{\text{str}}, \quad p_2|_{t=0} = P_{\text{str}}, \tag{3}$$

$$p_2|_{r=R_k} = P_{\text{str}}, \tag{4}$$

$$p_1|_{r=r_c} = \phi(t), \quad t < t_v, \tag{5}$$

$$\frac{\partial p_1}{\partial t} - C\frac{\partial p_1}{\partial r}\bigg|_{r=r_c} = 0, \quad t > t_v, \tag{6}$$

and sewing conditions at the interface:

$$p_1|_{r=R_1} = p_2|_{r=R_1}, \quad \chi_1 \frac{\partial p_1}{\partial r}\bigg|_{r=R_1} = \chi_2 \frac{\partial p_2}{\partial r}\bigg|_{r=R_1}. \tag{7}$$

Here

$$\chi = \frac{\sigma}{h\beta^*}, \quad \sigma(r) = \begin{cases} \sigma_1, & r_1 < r < R_1, \\ \sigma_2, & R_1 < r < R_k, \end{cases}$$

$$C = 2\pi\sigma(r_c) \cdot r_c \cdot \frac{\rho g \cos\alpha}{S} \quad \text{or} \quad C = 2\pi\sigma(r_c) \cdot \frac{\rho_c}{\beta_l V_p}$$

for modelling of IC or PRC, respectively.

The parameter of influence of the borehole C relates the rate of fluid inflow from the stratum to the rate of changing of the borehole pressure. In the first case (IC), the inflow of fluid from the stratum results in increasing of the fluid level in the borehole and in increasing of the hydrostatic pressure at the wellhead while in the second case, in increasing of the pressure because of compression of the fluid in a closed volume V_p. Detailed derivation of the boundary condition (6) can be found, e. g., in [7].

Other parameters are:

β^* — the elastocapacity of the stratum, Pa^{-1};

β_l — the compressibility of the fluid, Pa^{-1};

σ — the hydroconductivity, $\text{m}^3/(\text{Pa}\cdot\text{s})$;

χ — the piesoconductivity of the stratum, m^2/s; a piecewise-constant function χ models the circular near-borehole zone (NBZ) ($r_c < r < R_1$) with hydroconductivity σ_1 (piesoconductivity χ_1) and the stratum with hydroconductivity σ_2 (piesoconductivity χ_2);

h — the thickness of the stratum, m;

ρ — the density of the fluid in the borehole, kg/m^3, assumed to be constant;
atum, m;

ρ — the density of the fluid in the borehole, kg/m^3, assumed to be constant;

S — the area of cross-section of the fluid flux in the vicinity of the level, m^2;

α — the angle of borehole inclination from the vertical line in the interval of change of the level;

t_v — the duration of action at the stratum;

t_k — the moment when the process is stopped;

$\phi(t)$ — the change of the wellhead pressure during the interval t_v when the stratum is disturbed.

Given additional information, it is required to determine one or several from the following parameters: the stratum pressure P_{str}, the NBZ hydroconductivity σ_1, the stratum hydroconductivity σ_2, the radius of the near-borehole zone R_1.

The classical scheme of solution of such inverse problems is as follows [3]:

1) an assumption about the stratum structure, i. e., about the values of parameters, is made;
2) the problem (1) – (7) is solved numerically;
3) the trace of solution of such problem at the point $r = r_c$ is calculated;
4) the deviation of this trace from the given additional information $P_{\text{exp}}(r_c, t)$, $t_v < t < t_k$, is calculated;
5) new values of the stratum parameters that are to yield smaller deviation of data on step 4 are chosen, etc.

In other words, the algorithm requires multiple solution of the "direct problem". This circumstance leads to high "computational cost" of most of inverse problems whose solution by item-by-item examination of all possible values of the sought parameters is impossible (or inexpedient) [1, 2].

2. Solution of the direct problem

The direct problem is to find the pressure field $p_i(r, t)$ as solution of the problem (1) – (7) with known values of the stratum parameters $(\sigma_1, \sigma_2, R_1)$.

For its numerical solution, an algorithm realizing the finite difference method on an irregular (relative to r) mesh (the implicit scheme) was used. The program was written on Pascal and C++, tested, and included into the software complex for solution of the inverse problem.

For numerical experiments, the compiler Watcom C++ version 10.0A was used. Calculations were carried out with a Pentium-type processor Celeron 466. Solution of one direct problem for given parameters σ_1, σ_2, and R_1 takes 15 seconds.

The following values of the parameters were chosen:

$0 < \sigma_1, \sigma_2 < 100$ D $\cdot$ cm/cP, 1 D $= 10^{-12}$ m^2, 1 cP (santipoise) $= 10^{-3}$ Pa $\cdot$ s,
1 D $\cdot$ cm/cP $= 10^{-11}$ m^3/(Pa $\cdot$ s);

$r_c < R_1 < R_k$;

$\beta^* = (0.2 \cdot 4.5 + 1.35) \cdot 10^{-10}$ Pa$^{-1} = 2.25 \cdot 10^{-10}$ Pa^{-1}, the elastocapacity of the stratum;

$h = 5$ m, the thickness of the stratum;

$\rho = 1000$ kg/m^3, the density of the fluid;

$S = 2921$ mm$^2 = 2921 \cdot 10^{-6}$ m^2, the cross-section area of the fluid flux;

$\alpha = 0$;

$t_v = 60$ min, the duration of action at the stratum;

$t_k = 600$ min, the moment when the process is stopped;

$r_c = 108$ mm;

$R_k = 100$ m;

$P_{\text{str}} = 100$ atm.

The model function $P_{\text{exp}}(r_c, t)$ shows the change of the wellhead pressure. Recall that this function is the source data for the inversion algorithm that reconstructs the values of the desired coefficients.

3. Solution of the inverse problem

In order to reveal the peculiarities of the statement we preliminarily considered the following simplified statement:

Inverse problem 1. *Find the NBZ hydroconductivity σ_1 and the radius of the near-borehole zone R_1 if the hydroconductivity σ_2 of the remote zone of the stratum and the stratum pressure $P_{\rm str}$ are known.*

For solution of this and other inverse problems, the following objective functional was considered:

$$\Phi_1[R_1, \sigma_1] = \int\limits_{t_v}^{t_k} ||P_{\rm exp}(r_c, t) - P_{\rm dir}(r_c, t)||^2\, dt, \tag{8}$$

where $P_{\rm exp}(r_c, t)$ are the input data and $P_{\rm dir}(r_c, t)$ is the solution of the direct problem for current (test) values R_1 and σ_1.

At first the search was carried out over the mesh in large intervals of the parameter values and with a relatively large step, and then the search in the above-mentioned limits and with smaller step was carried out. The above intervals for small-step examination, which contain the true values of the sought parameters, were chosen with account of solution of the third inverse problem; in that solution, the values of σ_1 and R_1 were determined. The search was carried out in order to reveal structural peculiarities of the objective functional which later were taken into consideration in construction of a more sophisticated inversion algorithm that isn't based on exhaustive search.

The minimum of the functional lies in a comparatively narrow "ravine" that also contains a considerable number of local minima. This fact embarrasses application of standard optimization procedures.

The experience of solution of a number of inverse problems connected with exploration geophysics shows that the objective functional usually has a simpler structure in the case of "fixed geometry" of the medium, i. e., when the points of jump-like change of its parameters are given and only the medium parameters are to be found [1], [2]. Therefore, the following inverse problem was investigated:

Inverse problem 2. *Determine the NBZ hydroconductivity σ_1 and the stratum hydroconductivity σ_2 if the radius R_1 of the near-borehole zone is known.*

The following method was used for solution of this inverse problem: for fixed R_1, the values of σ_1 and σ_2 that yield the minimum of the objective functional

$$\Phi_2[\sigma_1, \sigma_2] = \int\limits_{t_v}^{t_k} ||P_{\rm exp}(r_c, t) - P_{\rm dir}(r_c, t)[\sigma_1, \sigma_2]||^2\, dt$$

are sought by the method of steepest descent [6].

The gradient of the objective functional was approximated by finite differences as follows:

$$\nabla_{\sigma_1}\Phi_2[\sigma_1, \sigma_2] = \frac{\Phi_2[\sigma_1 + \Delta\sigma_1, \sigma_2] - \Phi_2[\sigma_1 - \Delta\sigma_1, \sigma_2]}{2\Delta\sigma_1},$$

$$\nabla_{\sigma_2}\Phi_2[\sigma_1, \sigma_2] = \frac{\Phi_2[\sigma_1, \sigma_2 + \Delta\sigma_2] - \Phi_2[\sigma_1, \sigma_2 - \Delta\sigma_2]}{2\Delta\sigma_2}.$$

As was expected, the structure of the functional turned out to be much simpler than in the case of inverse problem 1. The ravine has no local minima and the minimum point is determined with high accuracy.

Finally, the complete statement was considered:

Inverse problem 3. *It is required to determine the NBZ hydroconductivity σ_1, the stratum hydroconductivity σ_2, and the radius of the near-borehole zone R_1 simultaneously.*

The following algorithm was used for its solution. At first, an interval of change of the parameter R_1 was chosen: $R_1^{\rm beg} \ldots R_1^{\rm end}$. Then for each fixed R_1^i from this interval, σ_1 and σ_2 which yield the minimum of the objective functional

$$\Phi_3[\sigma_1, \sigma_2, R_1] = \int\limits_{t_v}^{t_k} ||P_{\rm exp}(r_c, t) - P_{\rm dir}(r_c, t)[\sigma_1, \sigma_2, R_1]||^2\, dt$$

were sought.

The desired σ_1 and σ_2 were sought by the method of steepest descent. However, after we had got into the "ravine", we could not succeed in finding the direction of the gradient of the functional with sufficient accuracy (it was due to the fact that the gradient was approximated by finite differences), therefore, the descent to the point of minimum took too much time.

In order to avoid such difficulties, the following method was proposed. The following minimizing functions were introduced:

$$\begin{aligned} S_{\sigma_1}(\sigma_2, R_1)[N^G_{\sigma_1}] &= \{\sigma_1^* : \min_{\sigma_1^*} \Phi_3[\sigma_1^*, \sigma_2, R_1]\}, \\ S_{\sigma_2}(\sigma_1, R_1)[N^G_{\sigma_2}] &= \{\sigma_2^* : \min_{\sigma_2^*} \Phi_3[\sigma_1, \sigma_2^*, R_1]\}, \\ S_{R_1}(\sigma_1, \sigma_2)[N^G_{R_1}] &= \{R_1^* : \min_{R_1^*} \Phi_3[\sigma_1, \sigma_2, R_1^*]\}, \end{aligned}$$

i. e., these functions give the value of the parameter (σ_1, σ_2 or R_1) that minimizes the functional Φ_3 when other two parameters are fixed. Minimization was carried out by the method of "golden section" [6]; the parameter N^G_* specified the number of iterations in the search for the minimum.

4. Conclusion

In the frames of the inverse problem that is being investigated the authors are planning to carry out the following studies:

- to justify the correctness of statements of the direct and inverse problems;
- to study the questions of uniqueness of the solution;
- to obtain the estimates of conditional stability of the solution;
- to study the gradients and Hessians of the objective functionals (derivation of analytic formulas, the Fréchet differentiability, uniqueness of the minimum point);
- to study the possibility of application of regularization and combined (complex) objective functionals;
- to optimize the algorithm of inversion by using semi-analytic methods;
- to test the algorithm for its sensibility to errors (noise) in the data of inversion;
- to test the algorithm on data of actual field measurements.

References

[1] Alekseev A. S., Avdeev A. V., Fatianov A. G. and Cheverda V. A. Wave processes in vertically inhomogeneous media: a new strategy for a velocity inversion. Inverse Problems. 1993. Vol. 9, No. 3.

[2] Avdeev A. V., Goryunov E. V., Lavrentiev M. M., Jr. and Spigler R. Simultaneous identification of two coefficients in a diffusion equation. Preprint TR-8/99. Institute of Industrial and Applied Mathematics. Venice, Italy, 1999. 22 p.

[3] Avdeev A. V., Lavrentiev M. M., Jr. and Priimenko V. I. Inverse Problems and Some Applications. ICM&MG, Novosibirsk, 1999.

[4] Basniev K. S., Kochina I. N. and Maksimov V. M. Underground Hydromechanics. Nedra, Moscow, 1993 (in Russian).

[5] Buzinov S. N. and Umrikhin I. D. Investigation of Oil and Gas Wells and Strata. Nedra, Moscow, 1984 (in Russian).

[6] Vasil'ev F. P. Methods of Solution of Extremal Problems. Nauka, Moscow, 1981 (in Russian).

[7] Vljushin V.E. Exact solution of the problem of reconstruction of pressure in a well. Trudy MINH im. Gubkina. 1977. Vol. 129. P. 124–126 (in Russian).

[8] Zelenyak T. I., Lavrentiev M. M., Jr. and Vishnevskii M. P. Qualitative Theory of Parabolic Equations. Part I. VSP, Utrecht. The Netherlands, 1997.